MAD2104
Suggested problems on Chapter 2 material
(Sets)

1. List the members of these sets.
a. $\{x \mid x \text{ is a real number such that } x^2 = 1\}$
b. $\{x \mid x \text{ is a positive integer less than } 12\}$
c. $\{x \mid x \text{ is the square of an integer and } x < 100\}$
d. $\{x \mid x \text{ is an integer such that } x^2 = 2\}$

2. Determine whether each of these pairs of sets are equal.
a. $\{1, 3, 3, 3, 5, 5, 5, 5, 5\}$ and $\{5, 3, 1\}$
b. $\{\{1\}\}$ and $\{1, \{1\}\}$
c. $\emptyset$ and $\{\emptyset\}$

3. For each of the following sets, determine whether 2 is an element of that set.
a. $\{x \in \mathbb{R} \mid x \text{ is an integer greater than } 1\}$
b. $\{x \in \mathbb{R} \mid x \text{ is the square of an integer}\}$
c. $\{2, \{2\}\}$
d. $\{\{2\}, \{\{2\}\}\}$
e. $\{\{2\}, \{2, \{2\}\}\}$
f. $\{\{\{2\}\}\}$

4. For each of the sets in Question 3, determine whether $\{2\}$ is an element of that set.

5. Determine whether these statements are true or false.
a. $\emptyset \in \{\emptyset\}$
b. $\emptyset \in \{\emptyset, \{\emptyset\}\}$
c. $\{\emptyset\} \in \{\emptyset\}$
d. $\{\emptyset\} \in \{\{\emptyset\}\}$
e. $\{\emptyset\} \subset \{\emptyset, \{\emptyset\}\}$
f. $\{\{\emptyset\}\} \subset \{\emptyset, \{\emptyset\}\}$
g. $\{\{\emptyset\}\} \subset \{\{\emptyset\}, \{\emptyset\}\}$

6. Let $A = \{a, b, c, d\}$ and $B = \{y, z\}$. Find $A \times B$ and $B \times A$.

7. Translate each of these quantifications into English and determine its truth value.
a. $\forall x \in \mathbb{R}\ (x^2 \neq -1)$
b. $\exists x \in \mathbb{Z}\ (x^2 = 2)$
c. $\forall x \in \mathbb{Z}\ (x^2 > 0)$
d. $\exists x \in \mathbb{R}\ (x^2 = x)$
e. $\exists x \in \mathbb{R}\ (x^3 = -1)$
f. $\exists x \in \mathbb{Z}\ (x + 1 > x)$
g. $\forall x \in \mathbb{Z}\ (x - 1 \in \mathbb{Z})$
h. $\forall x \in \mathbb{Z}\ (x^2 \in \mathbb{Z})$

8. Let $A = \{1, 2, 3, 4, 5\}$ and $B = \{0, 3, 6\}$. Find the following.
a. $A \cup B$
b. $A \cap B$
c. $A - B$
d. $B - A$

9. Let A, B, and C be sets. Show the following.
a. $(A \cup B) \subseteq (A \cup B \cup C)$
b. $(A \cap B \cap C) \subseteq (A \cap B)$
c. $(A - B) - C \subseteq A - C$
d. $(A - C) \cap (C - B) = \emptyset$
e. $(B - A) \cup (C - A) = (B \cup C) - A$

10. Determine whether each of these functions from $\{a, b, c, d\}$ to itself is one-to-one.
a. $f(a) = b,\ f(b) = a,\ f(c) = c,\ f(d) = d$
b. $f(a) = b,\ f(b) = b,\ f(c) = d,\ f(d) = c$
c. $f(a) = d,\ f(b) = b,\ f(c) = c,\ f(d) = d$

11. Determine whether each of these functions from $\mathbb{Z}$ to $\mathbb{Z}$ is one-to-one.
a. $f(n) = n - 1$
b. $f(n) = n^2 + 1$
c. $f(n) = n^3$
d. $f(n) = \lceil n/2 \rceil$

12. Which functions in Question 11 are onto?

13. Find the terms of the sequence $\{a_n\}$, where $a_n = 2 \cdot (-3)^n + 5^n$.
a. a_0
b. a_1
c. a_4
d. a_5

14. Find the first five terms of the sequence defined by each of these recurrence relations and initial conditions.
a. $a_n = 6a_{n-1}, \qquad a_0 = 2$
b. $a_n = a_{n-1}^2, \qquad a_1 = 2$
c. $a_n = a_{n-1} + 3a_{n-2}, \qquad a_0 = 1, \qquad a_1 = 2$
d. $a_n = na_{n-1} + n^2a_{n-2}, \qquad a_0 = 1, \qquad a_1 = 1$
e. $a_n = a_{n-1} + a_{n-3}, \qquad a_0 = 1, \qquad a_1 = 2, \qquad a_2 = 0$

15. Let $a_n = 2^n + 5 \cdot 3^n$ for $n = 0, 1, 2, \ldots$.
a. Find a_0, a_1, a_2, a_3, and a_4.
b. Show that $a_2 = 5a_1 - 6a_0$, $a_3 = 5a_2 - 6a_1$, and $a_4 = 5a_3 - 6a_2$.
c. Show that $a_n = 5a_{n-1} - 6a_{n-2}$ for all integers n with $n \geq 2$.

16. Determine whether each of these sets is finite, countably infinite, or uncountable. For those that are countably infinite, exhibit a one-to-one correspondence between the set of positive integers and that set.
a. the negative integers
b. the even integers
c. the integers less than 100
d. the real numbers between 0 and $\frac{1}{2}$
e. the positive integers less than 1,000,000,000
f. the integers that are multiples of 7

17. Determine whether each of these sets is countable or uncountable. For those that are countably infinite, exhibit a one-to-one correspondence between the set of positive integers and that set.
a. all bit strings not containing the bit 0
b. all positive rational numbers that cannot be written with denominators less than 4
c. the real numbers not containing 0 in their decimal representation
d. the real numbers containing only a finite number of 1s in their decimal representation

18. Find **AB** if
a. $\mathbf{A} = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} 0 & 4 \\ 1 & 3 \end{bmatrix}$
b. $\mathbf{A} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \\ 2 & 3 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} 3 & -2 & -1 \\ 1 & 0 & 2 \end{bmatrix}$
c. $\mathbf{A} = \begin{bmatrix} 4 & -3 \\ 3 & -1 \\ 0 & -2 \\ -1 & 5 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} -1 & 3 & 2 & -2 \\ 0 & -1 & 4 & -3 \end{bmatrix}$