

MAC2233

Suggested problems on Chapter 1 material (functions, graphs, limits, continuity)

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1. Find the limit, if it exists.

$$\lim_{x \rightarrow 3} \frac{2x + 3}{x - 3}$$

2. Find the limit, if it exists.

$$\lim_{x \rightarrow 3} \frac{9 - x^2}{x - 3}$$

3. Find the limit, if it exists.

$$\lim_{x \rightarrow 4} \frac{\sqrt{x} - 2}{x - 4}$$

4. Find $\lim_{x \rightarrow +\infty} f(x)$ and $\lim_{x \rightarrow -\infty} f(x)$. If the limiting value is infinite, indicate whether it is $+\infty$ or $-\infty$.

$$f(x) = \frac{1 - 3x^3}{2x^3 - 6x + 2}$$

5. Find $\lim_{x \rightarrow +\infty} f(x)$ and $\lim_{x \rightarrow -\infty} f(x)$. If the limiting value is infinite, indicate whether it is $+\infty$ or $-\infty$.

$$f(x) = \frac{x^2 + x - 5}{1 - 2x - x^3}$$

6. Find $\lim_{x \rightarrow +\infty} f(x)$ and $\lim_{x \rightarrow -\infty} f(x)$. If the limiting value is infinite, indicate whether it is $+\infty$ or $-\infty$.

$$f(x) = \frac{1 - 2x^3}{x + 1}$$

7. A business manager determines that t months after production begins on a new product, the number of units produced will be P thousand, where

$$P(t) = \frac{6t^2 + 5t}{(t + 1)^2}$$

What happens to production in the long run (as $t \rightarrow \infty$)?

8. Find the indicated one-sided limit. If the limiting value is infinite, indicate whether it is $+\infty$ or $-\infty$.

$$\lim_{x \rightarrow 2^-} \frac{x^2 + 4}{x - 2}$$

9. Find $\lim_{x \rightarrow 3^-} f(x)$ and $\lim_{x \rightarrow 3^+} f(x)$, where

$$f(x) = \begin{cases} 2x^2 - x & \text{if } x < 3 \\ 3 - x & \text{if } x \geq 3 \end{cases}$$

10. List all the values of x for which the given function is not continuous.

$$f(x) = \frac{x}{x^2 - x}$$

11. List all the values of x for which the given function is not continuous.

$$f(x) = \begin{cases} 3x - 2 & \text{if } x < 0 \\ x^2 + x & \text{if } x \geq 0 \end{cases}$$

12. In 2010, the cost $p(x)$ in cents of mailing a letter weighing x ounces was

$$p(x) = \begin{cases} 44 & \text{if } 0 < x \leq 1 \\ 61 & \text{if } 1 < x \leq 2 \\ 78 & \text{if } 2 < x \leq 3.5 \end{cases}$$

Sketch the graph of $p(x)$ for $0 < x \leq 3.5$. For which of those values of x is $p(x)$ discontinuous?

13. Find the value of the constant A so that the function $f(x)$ will be continuous for all x .

$$f(x) = \begin{cases} Ax - 3 & \text{if } x < 2 \\ 3 - x + 2x^2 & \text{if } x \geq 2 \end{cases}$$

14. Find all the indicated (one-sided and two-sided) limits, or state that they do not exist.

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|------------------------------------|------------------------------------|----------------------------------|
| a. $\lim_{x \rightarrow 0^-} f(x)$ | b. $\lim_{x \rightarrow 0^+} f(x)$ | c. $\lim_{x \rightarrow 0} f(x)$ |
| d. $\lim_{x \rightarrow 2^-} f(x)$ | e. $\lim_{x \rightarrow 2^+} f(x)$ | f. $\lim_{x \rightarrow 2} f(x)$ |
| g. $\lim_{x \rightarrow 4^-} f(x)$ | h. $\lim_{x \rightarrow 4^+} f(x)$ | i. $\lim_{x \rightarrow 4} f(x)$ |

