

MAC2233  
MORE suggested problems on Chapter 2 material  
(differentiation)

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1. Find  $\frac{dy}{dx}$  if  $y = u^3 + u$  and  $u = \frac{1}{\sqrt{x}}$ . Simplify your answer.
2. If  $y = \frac{1}{u+1}$  and  $u = x^3 - 2x + 5$ , find the value of  $\frac{dy}{dx}$  at  $x = 0$ .
3. Differentiate the function and simplify your answer.

$$g(x) = \frac{1}{\sqrt{4x^2 + 1}}$$

4. Differentiate the function and simplify your answer.

$$h(s) = (1 + \sqrt{3s})^5$$

5. Differentiate the function and simplify your answer.

$$g(x) = \sqrt{1 + \frac{1}{3x}}$$

6. Differentiate the function and simplify your answer.

$$f(x) = 2(3x + 1)^4(5x - 3)^2$$

7. Differentiate the function and simplify your answer.

$$f(x) = \frac{(x + 1)^5}{(1 - x)^4}$$

8. Find the second derivative of the function.

$$y = (1 - 2x^3)^4$$

9. At a certain factory, the total cost of manufacturing  $q$  units is  $C(q) = 0.2q^2 + q + 900$  dollars. It has been determined that approximately  $q(t) = t^2 + 100t$  units are manufactured during the first  $t$  hours of a production run. Compute the rate at which the total manufacturing cost is changing with respect to time 1 hour after production begins.

10. The number of units  $Q$  of a particular commodity that will be produced when  $L$  worker-hours of labor are employed is modeled by

$$Q(L) = 300L^{1/3}$$

Suppose that the labor level varies with time in such a way that  $t$  months from now  $L(t)$  worker-hours will be employed, where

$$L(t) = \sqrt{739 + 3t - t^2}$$

for  $0 \leq t \leq 12$ .

(a) How many worker-hours will be employed in producing the commodity 5 months from now? How many units will be produced at this time?

(b) At what rate will production be changing with respect to time 5 months from now? Will production be increasing or decreasing at this time?

11. Suppose the total cost in dollars of manufacturing  $q$  units is  $C(q) = 3q^2 + q + 500$ .

(a) Use marginal analysis to estimate the cost of manufacturing the 41st unit.

(b) Compute the actual cost of manufacturing the 41st unit.

12. At a certain factory, the daily output is  $Q(K) = 600K^{1/2}$  units, where  $K$  denotes the capital investment measured in units of \$1,000. The current capital investment is \$900,000. Estimate the effect that an additional capital investment of \$800 will have on the daily output.

13. At a certain factory, the daily output is  $Q(L) = 60,000L^{1/3}$  units, where  $L$  denotes the size of the labor force measured in worker-hours. Currently 1,000 worker-hours of labor are used each day. Estimate the effect on output that will be produced if the labor force is cut to 940 worker-hours.

14. A projection made in January of 2005 determined that  $x$  years later, the average property tax on a three-bedroom home in a certain community will be  $T(x) = 60x^{3/2} + 40x + 1200$  dollars. Estimate the percentage change by which the property tax will increase during the first half of the year 2013.

15. It is projected that  $t$  years from now, the circulation of a local newspaper will be  $C(t) = 100t^2 + 400t + 5000$ . Estimate the amount by which the circulation will increase during the next 6 months.