MAC2233
Suggested problems on Chapter 2 material (differentiation)

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1. Compute the derivative of the given function and find the slope of the line that is tangent to its graph for the specified value of the independent variable.

$$f(x) = 2x^2 - 3x - 5, \qquad x = 0$$

2. Compute the derivative of the given function and find the equation of the line that is tangent to its graph for the specified value $x = c$.

$$f(x) = \frac{-2}{x}, \qquad c = -1$$

3. Compute the derivative of the given function and find the equation of the line that is tangent to its graph for the specified value $x = c$.

$$f(x) = 2\sqrt{x}, \qquad c = 4$$

4. Let $s(t) = \sqrt{t}$.

(a) Find the average rate of change of $s(t)$ with respect to t as t changes from $t = 1$ to $t = \frac{1}{4}$.

(b) Use calculus to find the instantaneous rate of change of $s(t)$ at $t = 1$, and compare with the average rate found in part (a).

5. First compute the derivative of $f(x) = x^3$ and then use it to find the slope of the tangent line to the curve $y = x^3$ at the point where $x = -1$. What is the equation of the tangent line at this point?

6. First compute the derivative of $f(x) = \sqrt{x}$ and then use it to:

(a) Find the equation of the tangent line to the curve $y = \sqrt{x}$ at the point where $x = 4$.

(b) Find the rate at which $y = \sqrt{x}$ is changing with respect to x when $x = 1$.

7. Differentiate the given function.

$$f(x) = \frac{1}{4}x^8 - \frac{1}{2}x^6 - x + 2$$

8. Differentiate the given function.

$$y = \frac{3}{x} - \frac{2}{x^2} + \frac{2}{3x^3}$$

9. Differentiate the given function.

$$y = -\frac{x^2}{16} + \frac{2}{x} - x^{3/2} + \frac{1}{3x^2} + \frac{x}{3}$$

10. Differentiate the given function

$$y = \frac{x^5 - 4x^2}{x^3} \qquad [\textit{Hint:} \text{ Divide first.}]$$

11. Differentiate the given function

$$y = x^2(x^3 - 6x + 7) \qquad [\textit{Hint:} \text{ Multiply first.}]$$

12. Find the equation of the line that is tangent to the graph of the given function at the given point.

$$y = \sqrt{x^3} - x^2 + \frac{16}{x^2}, \qquad (4, -7)$$

13. Find the equation of the line that is tangent to the graph of the given function at the given point.

$$y = (x^2 - x)(3 + 2x), \qquad (-1, 2)$$

14. Find the rate of change of the given function $f(x)$ with respect to x for the prescribed value $x = c$.

$$f(x) = \frac{2}{x} - x\sqrt{x}, \qquad x = 1$$

15. The gross annual earnings of a certain company were $A(t) = 0.1t^2 + 10t + 20$ thousand dollars t years after its formation in 2008.

(a) At what rate were the gross annual earnings growing with respect to time in 2012?

(b) At what percentage rate were the gross annual earnings growing with respect to time in 2012?

16. Records indicate that x years after 2008, the property tax on a three-bedroom home in a certain community was $T(x) = 20x^2 + 40x + 600$ dollars.

(a) At what rate was the property tax increasing with respect to time in 2008?

(b) By how much did the tax change between 2008 and 2012?

17. After x weeks, the number of people using a new rapid transit system was $N(x) = 6x^3 + 500x + 8000$.

(a) At what rate was the use of the system changing with respect to time after 8 weeks?

(b) By how much did the use of the system change during the eighth week?

18. It is projected that x months from now, the population of a certain town will be $P(x) = 2x + 4x^{3/2} + 5000$.

(a) At what rate will the population be changing with respect to time 9 months from now?

(b) At what percentage rate will the population be changing with respect to time 9 months from now?

19. Differentiate the given function.

$$f(x) = \frac{x^2 - 3x + 2}{2x^2 + 5x - 1}$$

20. Find an equation for the tangent line to the given curve at the point where $x = x_0$.

$$y = (x^2 + 3x - 1)(2 - x), \qquad x_0 = 1$$

21. Find all points on the graph of the given function where the tangent line is horizontal.

$$f(x) = \frac{x + 1}{x^2 + x + 1}$$

22. Find the rate of change $\dfrac{dy}{dx}$ for the prescribed value of x_0.

$$y = (x^2 + 2)(x + \sqrt{x}), \qquad x_0 = 4$$

23. Find the second derivative of the given function. Use the appropriate notation for the second derivative and simplify your answer. (Don't forget to simplify the first derivative as much as possible before computing the second derivative.)

$$y = (x^2 - x)\left(2x - \frac{1}{x}\right)$$

24. An efficiency study of the morning shift at a certain factory indicates that an average worker arriving on the job at 8:00 a.m. will have produced $Q(t) = -t^3 + 8t^2 + 15t$ units t hours later.

(a) Compute the worker's rate of production $R(t) = Q'(t)$.

(b) At what rate is the worker's rate of production changing with respect to time at 9:00 a.m.?

25. It is estimated that t years from now, the population of a certain suburban community will be $P(t) = 20 - \dfrac{6}{t+1}$ thousand.

(a) Derive a formula for the rate at which the population will be changing with respect to time t years from now.

(b) At what rate will the population be growing 1 year from now?

(c) By how much will the population actually increase during the second year?

(d) At what rate will the population be growing 9 years from now?

(e) What will happen to the rate of population growth in the long run?