$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$$

19. Differentiate the given function.

$$f(x) = \frac{x^2 - 3x + 2}{2x^2 + 5x - 1}$$

$$f'(x) = \frac{(x^2-3x+2)'(2x^2+5x-1) - (x^2-3x+2)(2x^2+5x-1)'}{(2x^2+5x-1)^2}$$

$$= \frac{(2x-3)(2x^2+5x-1) - (x^2-3x+2)(4x+5)}{(2x^2+5x-1)^2}$$

Now you could expand the top, but that's a lot of writing.
You don't have to in this problem.
HOWEVER, it's sometimes good to practice your algebra.

$$(2x-3)(2x^2+5x-1) = 4x^3 + 10x^2 - 2x - 6x^2 - 15x + 3$$
$$= 4x^3 + 4x^2 - 17x + 3$$

$$(x^2-3x+2)(4x+5) = 4x^3 + 5x^2 - 12x^2 - 15x + 8x + 10$$
$$= 4x^3 - 7x^2 - 7x + 10$$

So numerator is $(4x^3+4x^2-17x+3) - (4x^3-7x^2-7x+10)$

$= 4x^3 + 4x^2 - 17x + 3 - 4x^3 + 7x^2 + 7x - 10$

$= 11x^2 - 10x - 7$. Final answer is $\dfrac{11x^2-10x-7}{(2x^2+5x-1)^2}$

20. Find an equation for the tangent line to the given curve at the point where $x = x_0$.

$$y = (x^2 + 3x - 1)(2 - x), \qquad x_0 = 1$$

First expand.

$$y = 2x^2 - x^3 + 6x - 3x^2 - 2 + x$$

$$y = -x^3 - x^2 + 7x - 2$$

$$\frac{dy}{dx} = y' = -3x^2 - 2x + 7$$

At $x = x_0 = 1$, slope $= \frac{dy}{dx} = -3 \cdot 1^2 - 2 \cdot 1 + 7$
$= -3 - 2 + 7 = 2$

At $x = x_0 = 1$, what is y?

We have $y = (1^2 + 3 \cdot 1 - 1)(2 - 1)$
$= (1 + 3 - 1) \cdot 1 = 3$

The slope is 2 and the point is $(1, 3)$

An equation for the tangent line is

$$y - 3 = 2(x - 1)$$

or $y - 3 = 2x - 2$

or $y = 2x + 1$

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$$

21. Find all points on the graph of the given function where the tangent line is horizontal.

$$f(x) = \frac{x+1}{x^2+x+1}$$

$$f'(x) = \frac{(x+1)'(x^2+x+1) - (x+1)(x^2+x+1)'}{(x^2+x+1)^2}$$

$$= \frac{1(x^2+x+1) - (x+1)(2x+1)}{(x^2+x+1)^2} = \frac{(x^2+x+1) - (2x^2+3x+1)}{(x^2+x+1)^2}$$

$$= \frac{x^2+x+1-2x^2-3x-1}{(x^2+x+1)^2} = \frac{-x^2-2x}{(x^2+x+1)^2}$$

Now, tangent line is horizontal if $f'(x) = 0$, which happens if numerator of f' is zero.

$-x^2 - 2x = 0$ $\qquad \rightarrow x = 0$ or $x = -2$

$-x(x+2) = 0$ $\qquad$ What are the points?

If $x=0$ then $y = \frac{0+1}{0^2+0+1} = \frac{1}{1} = 1$. Point $(0,1)$

If $x=-2$ then $y = \frac{-2+1}{4-2+1} = \frac{-1}{3}$. Point $\left(-2, -\frac{1}{3}\right)$.

22. Find the rate of change $\frac{dy}{dx}$ for the prescribed value of x_0.

$$y = (x^2 + 2)(x + \sqrt{x}), \qquad x_0 = 4$$

$$y = (x^2+2)(x+x^{1/2})$$

$$y = x^3 + x^{5/2} + 2x + 2x^{1/2}$$

$$\frac{dy}{dx} = y' = 3x^2 + \frac{5}{2}x^{3/2} + 2 + 2\cdot\frac{1}{2}x^{-1/2}$$

$$= 3x^2 + \frac{5}{2}x^{3/2} + 2 + \frac{1}{x^{1/2}}$$

If $x = x_0 = 4$, then

$$\frac{dy}{dx} = 3\cdot4^2 + \frac{5}{2}\cdot4^{3/2} + 2 + \frac{1}{4^{1/2}}$$

$$= 3\cdot16 + \frac{5}{2}\cdot2^3 + 2 + \frac{1}{2}$$

$$= 48 + 5\cdot4 + 2 + \frac{1}{2}$$

$$= 70 + \frac{1}{2} = \frac{140}{2} + \frac{1}{2} = \frac{141}{2}$$

23. Find the second derivative of the given function. Use the appropriate notation for the second derivative and simplify your answer. (Don't forget to simplify the first derivative as much as possible before computing the second derivative.)

$$y = (x^2 - x)\left(2x - \frac{1}{x}\right)$$

$$y = 2x^3 - x - 2x^2 + 1$$

$$y' = 6x^2 - 1 - 4x$$

$$y'' = 12x - 0 - 4 \quad = 12x - 4$$

24. An efficiency study of the morning shift at a certain factory indicates that an average worker arriving on the job at 8:00 a.m. will have produced $Q(t) = -t^3 + 8t^2 + 15t$ units t hours later.

(a) Compute the worker's rate of production $R(t) = Q'(t)$.

(b) At what rate is the worker's rate of production changing with respect to time at 9:00 a.m.?

(a) $R(t) = Q'(t) = -3t^2 + 8 \cdot 2t + 15 \cdot 1$

$= -3t^2 + 16t + 15$

(b) Notice this is asking:

At what rate is $R(t)$ changing? $R'(t) = Q''(t)$

$R'(t) = -3 \cdot 2t + 16 \cdot 1 + 0$

$= -6t + 16$

At 9:00 a.m. we have $t = 1$

$R'(1) = -6 \cdot 1 + 16 = -6 + 16 = 10$

(units per hour per hour)

25. It is estimated that t years from now, the population of a certain suburban community will be $P(t) = 20 - \frac{6}{t+1}$ thousand.

(a) Derive a formula for the rate at which the population will be changing with respect to time t years from now.

(b) At what rate will the population be growing 1 year from now?

(c) By how much will the population actually increase during the second year?

(d) At what rate will the population be growing 9 years from now?

(e) What will happen to the rate of population growth in the long run?

(a) $P(t) = 20 - 6(t+1)^{-1}$

$$P'(t) = 0 - 6\cdot(-1)(t+1)^{-2}\cdot 1 = \frac{6}{(t+1)^2}$$

(b) $P'(1) = \frac{6}{(1+1)^2} = \frac{6}{2^2} = \frac{6}{4} = \frac{3}{2}$ or 1.5 thousand per year

(c) $P(1) = 20 - \frac{6}{1+1} = 20 - \frac{6}{2} = 20 - 3 = 17$

$$P(2) = 20 - \frac{6}{2+1} = 20 - \frac{6}{3} = 20 - 2 = 18$$

During the second year, population increases by $18 - 17 = 1$ thousand

(d) $P'(9) = \frac{6}{(9+1)^2} = \frac{6}{10^2} = \frac{6}{100}$ or 0.06 thousand per year

(e) $\lim_{t\to\infty} P'(t) = \lim_{t\to\infty} \frac{6}{(t+1)^2} = 0$

In the long run, the rate of change of the population approaches zero