

MAC2233
Suggested problems on Chapter 3 material
(applications of the derivative)

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February 23, 2018

1. Find the intervals of increase and decrease for the given function.

$$f(t) = t^3 + 3t^2 + 1$$

2. Find the intervals of increase and decrease for the given function.

$$f(x) = 3x^5 - 5x^3$$

- 3.** Sketch a graph of a function that has all the following properties:
- a.** $f'(0) = f'(1) = f'(2) = 0$
 - b.** $f'(x) < 0$ when $x < 0$ and when $x > 2$
 - c.** $f'(x) > 0$ when $0 < x < 1$ and when $1 < x < 2$

4. Sketch a graph of a function that has all the following properties:
- a. $f'(0) = f'(1) = f'(2) = 0$
 - b. $f'(x) < 0$ when $0 < x < 1$
 - c. $f'(x) > 0$ when $x < 0$ and when $1 < x < 2$ and when $x > 2$

5. Sketch a graph of a function that has all the following properties:
- a. $f'(x) > 0$ when $x < -5$ and when $x > 1$
 - b. $f'(x) < 0$ when $-5 < x < 1$
 - c. $f(-5) = 4$ and $f(1) = -1$

6. Sketch a graph of a function that has all the following properties:
- a. $f'(x) < 0$ when $x < -1$
 - b. $f'(x) > 0$ when $-1 < x < 3$ and when $x > 3$
 - c. $f'(-1) = 0$ and $f'(3) = 0$

7. Determine where the function is increasing and decreasing, and where its graph is concave up and concave down. Find the relative extrema and inflection points, and sketch the graph of the function.

$$f(x) = x^3 + 3x^2 + 1$$

8. Determine where the function is increasing and decreasing, and where its graph is concave up and concave down. Find the relative extrema and inflection points, and sketch the graph of the function.

$$f(x) = x^4 - 4x^3 + 10$$

9. Determine where the function is increasing and decreasing, and where its graph is concave up and concave down. Find the relative extrema and inflection points, and sketch the graph of the function.

$$f(x) = x^3 - 3x^2 + 3x + 1$$

10. Find the relative maxima and minima of the function.

$$f(x) = 2x + 1 + \frac{18}{x}$$

11. Find all vertical and horizontal asymptotes of the graph of the function.

$$f(x) = \frac{x}{2-x}$$

12. Sketch the graph of the function.

$$f(x) = \frac{x+3}{x-5}$$

13. Sketch the graphs of $f(x) = x^{2/3}$ and $g(x) = (x - 1)^{1/3}$.

14. Find the absolute maximum and absolute minimum (if any) of the given function on the specified interval.

$$f(x) = x^2 + 4x + 5, \quad -3 \leq x \leq 1$$

15. Find the absolute maximum and absolute minimum (if any) of the given function on the specified interval.

$$f(x) = x^3 + 3x^2 + 1, \quad -3 \leq x \leq 2$$

16. Find the absolute maximum and absolute minimum (if any) of the given function on the specified interval.

$$f(x) = \frac{1}{3}x^3 - 9x + 2, \quad 0 \leq x \leq 2$$

17. Find the absolute maximum and absolute minimum (if any) of the given function on the specified interval.

$$f(x) = x^5 - 5x^4 + 1, \quad 0 \leq x \leq 5$$

18. Find the absolute maximum and absolute minimum (if any) of the given function on the specified interval.

$$f(t) = 3t^5 - 5t^3, \quad -2 \leq t \leq 0$$

19. Find the absolute maximum and absolute minimum (if any) of the given function on the specified interval.

$$f(x) = 10x^6 + 24x^5 + 15x^4 + 3, \quad -1 \leq x \leq 1$$

20. Find the absolute maximum and absolute minimum (if any) of the given function on the specified interval.

$$f(x) = (x^2 - 4)^5, \quad -3 \leq x \leq 2$$

21. Find the absolute maximum and absolute minimum (if any) of the given function on the specified interval.

$$f(t) = \frac{t^2}{t-1}, \quad -2 \leq t \leq -\frac{1}{2}$$

22. Find the absolute maximum and absolute minimum (if any) of the given function on the specified interval.

$$g(x) = x + \frac{1}{x}, \quad \frac{1}{2} \leq x \leq 3$$

23. Find the absolute maximum and absolute minimum (if any) of the given function on the specified interval.

$$g(x) = \frac{1}{x^2 - 9}, \quad 0 \leq x \leq 2$$

24. Suppose the price at which q units of a particular commodity can be sold is

$$p(q) = 49 - q$$

and suppose the total cost of producing the q units is

$$C(q) = \frac{1}{8}q^2 + 4q + 200.$$

- a.** Find the revenue function $R(q)$, the profit function $P(q)$, the marginal revenue $R'(q)$, and the marginal cost $C'(q)$. Sketch the graphs of $P(q)$, $R'(q)$, and $C'(q)$ on the same coordinate axes and determine the level of production q where $P(q)$ is maximized.
- b.** Find the average cost $A(q) = \frac{C(q)}{q}$, and sketch the graphs of $A(q)$ and the marginal cost $C'(q)$ on the same axes. Determine the level of production q at which $A(q)$ is minimized.

- 25.** Suppose the demand q and price p for a certain commodity are related by the linear equation $q = 240 - 2p$ (for $0 \leq p \leq 120$).
- Express the elasticity of demand as a function of p .
 - Calculate the elasticity of demand when the price is $p = 100$. Interpret your answer.
 - Calculate the elasticity of demand when the price is $p = 50$. Interpret your answer.
 - At what price is the elasticity of demand equal to 1? What is the economic significance of this price?

26. Compute the elasticity of demand for the given demand function $D(p)$ and determine whether the demand is elastic, inelastic, or of unit elasticity at the indicated price p .

$$D(p) = 200 - p^2, \quad p = 10$$

- 27.** An art gallery offers 50 prints by a famous artist. If each print in the limited edition is priced at p dollars, it is expected that $q = 500 - 2p$ prints will be sold.
- a.** What limitations are there on the possible range of the price p ?
 - b.** Find the elasticity of demand. Determine the values of p for which the demand is elastic, inelastic, and of unit elasticity.
 - c.** Interpret the results of part (b) in terms of the behavior of the total revenue as a function of unit price p .
 - d.** If you were the owner of the gallery, what price would you charge for each print? Explain the reasoning behind your decision.

28. A bookstore can obtain a certain novel from the publisher at a cost of \$3 per book. The bookstore has been offering the novel at a price of \$15 per copy and, at this price, has been selling 200 copies a month. The bookstore is planning to lower its price to stimulate sales and estimates that for each \$1 reduction in the price, 20 more books will be sold each month. At what price should the bookstore sell the novel to generate the greatest possible profit?

29. Each machine at a certain factory can produce 50 units per hour. The setup cost is \$80 per machine, and the operating cost is \$5 per hour. How many machines should be used to produce 8,000 units at the least possible cost? (Remember that the answer should be a whole number.)

30. It is estimated that the cost of constructing an office building that is n floors high is $C(n) = 2n^2 + 500n + 600$, measured in thousands of dollars. How many floors should the building have in order to minimize the average cost per floor? (Remember that your answer should be a whole number.)

31. A manufacturer of medical monitoring devices uses 36,000 cases of components per year. The ordering cost is \$54 per shipment, and the annual cost of storage is \$1.20 per case. The components are used at a constant rate throughout the year, and each shipment arrives just as the preceding shipment is being used up. How many cases should be ordered in each shipment to minimize total cost?

32. You are the manager of a company that manufactures bicycles, and you buy 6,000 tires a year from a distributor. Each tire costs \$21, the ordering fee is \$20 per shipment, and the storage cost is 96 cents per tire per year. Suppose that the tires are used at a constant rate throughout the year and that each shipment arrives just as the preceding shipment is being used up. How many tires should you order each time to minimize total cost?

33. A carpenter has been asked to build an open box with a square base. The sides of the box will cost \$3 per square meter, and the base will cost \$4 per square meter. What are the dimensions of the box of greatest volume that can be constructed for \$48?

34. Carla is a carpenter who has been hired to make a closed box with a square base and a volume of 250 cubic meters. The material for the top and bottom of the box costs \$2 per square meter, and the material for the sides costs \$1 per square meter. Can Carla construct the box for less than \$300?