29. Each machine at a certain factory can produce 50 units per hour. The setup cost is \$80 per machine, and the operating cost is \$5 per hour. How many machines should be used to produce 8,000 units at the least possible cost? (Remember that the answer should be a whole number.)

Let x = number of machines

Let t = operating time in hours

Want to minimize TOTAL COST. Total cost = setup cost + operating cost.

Setup cost = (80 dollars per machine)$\cdot$(x machines) = $80x$ dollars

Operating cost = (5 dollars per hour)$\cdot$(t hours) = $5t$ dollars

so total cost $= 80x + 5t$. THIS IS WHAT WE WANT TO MINIMIZE.

What else do we know? Total production must be 8000 units.

Total production = (50 units per machine per hour)$\cdot$(x machines)$\cdot$(t hours)

= $50xt$ units.

Want $50xt = 8000 \Rightarrow xt = \frac{8000}{50} = 160 \Rightarrow t = \frac{160}{x} = 160x^{-1}$

Want to minimize total cost $C = 80x + 5t$

$C(x) = 80x + 5\cdot 160x^{-1} = 80x + 800x^{-1}$

$C'(x) = 80 - 800x^{-2}$. Critical numbers? $80 - \frac{800}{x^2} = 0$

$\Rightarrow 80 = \frac{800}{x^2} \Rightarrow x^2 = 10$. Note: if x is small then $C' = 80 - \frac{800}{x^2}$ is negative.

If x is big then C' is positive.

So C is minimized when $x = \sqrt{10}$. However, we can't choose $x = \sqrt{10}$.

Closest possible is $x = 3$ or 4. $C(3) = 80\cdot 3 + 800\cdot\frac{1}{3} = 240 + 266.67$

$= 506.67$

$C(4) = 80\cdot 4 + 800\cdot\frac{1}{4} = 320 + 200 = 520$. We should use 3 machines.

30. It is estimated that the cost of constructing an office building that is n floors high is $C(n) = 2n^2 + 500n + 600$, measured in thousands of dollars. How many floors should the building have in order to minimize the average cost per floor? (Remember that your answer should be a whole number.)

Total cost $= C(n) = 2n^2 + 500n + 600$ (in thousands of dollars)

Average cost per floor $A = \dfrac{\text{total cost}}{\#\text{ of floors}} = \dfrac{2n^2 + 500n + 600}{n}$

$A(n) = 2n + 500 + 600n^{-1}$. Want to minimize $A(n)$.

$A'(n) = 2 + 0 - 600n^{-2} = 2 - \dfrac{600}{n^2}$

Critical numbers? $2 - \dfrac{600}{n^2} = 0 \Rightarrow 2 = \dfrac{600}{n^2}$

$\Rightarrow n^2 = 300 \Rightarrow n = \sqrt{300} \quad (= \sqrt{3} \cdot \sqrt{100} = 10\sqrt{3} \approx 17.32)$

Note: if n is small (say $n=1$) then $A' = 2 - \dfrac{600}{n^2}$ is negative

If n is big then $A' = 2 - \dfrac{600}{n^2}$ is positive (say $n=1000$)

So $A(n)$ is minimized when $n = \sqrt{300}$. But n must be a whole number. The closest whole numbers are 17 and 18.

$A(17) = 2 \cdot 17 + 500 + \dfrac{600}{17} =$ (using calculator) $= 569.29$

$A(18) = 2 \cdot 18 + 500 + \dfrac{600}{18} =$ (using calculator) $= 569.33$

It's ridiculously close, but we should choose $n = 17$ floors if we want to minimize average cost per floor.

31. A manufacturer of medical monitoring devices uses 36,000 cases of components per year. The ordering cost is \$54 per shipment, and the annual cost of storage is \$1.20 per case. The components are used at a constant rate throughout the year, and each shipment arrives just as the preceding shipment is being used up. How many cases should be ordered in each shipment to minimize total cost?

Let x = number of cases ordered per shipment.

Want to minimize total cost.

Total cost = (cost of ordering) + (cost of storage).

Maybe let y = number of shipments per year.

Must use 36,000 cases per year.

So, (# of cases per shipment) • (# of shipments per year) = 36,000.

That is, $xy = 36{,}000$ so $y = \dfrac{36{,}000}{x}$.

Ordering cost = (54 dollars per shipment) • (# of shipments) = $54y$

Storage cost = (1.2 dollars per case) • (# of cases stored) = $1.2x$

Total cost $= 54y + 1.2x = 54 \cdot \dfrac{36{,}000}{x} + 1.2x$ THIS IS WHAT WE WANT TO MINIMIZE

$C = 54 \cdot 36{,}000x^{-1} + 1.2x = 1{,}944{,}000x^{-1} + 1.2x$

$C'(x) = -1{,}944{,}000x^{-2} + 1.2$. Critical numbers?

$$\frac{-1{,}944{,}000}{x^2} + 1.2 = 0$$

$$1.2 = \frac{1{,}944{,}000}{x^2}$$

$$1.2x^2 = 1{,}944{,}000$$

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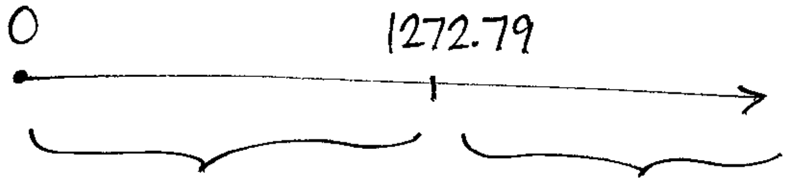
$$x^2 = \frac{1,944,000}{1.2} = 1,620,000$$

$$x = \sqrt{1,620,000} = 1272.79$$



If $x = 1$ (e.g.) then $C' = \dfrac{-1944000}{x^2} + 1.2$ is negative

C decreasing

If $x = 1000000$ (e.g.) then $C' = \dfrac{-1944000}{x^2} + 1.2$ is positive

C is increasing

C is MINIMIZED if $x = 1272.79$

x must be a whole number so try $x = 1272$, $x = 1273$

$$C(1272) = \frac{1,944,000}{1272} + 1.2 \cdot 1272 = 3054.7018$$

$$C(1273) = \frac{1,944,000}{1273} + 1.2 \cdot 1273 = 3054.7013$$

To minimize total cost, the number of cases per shipment should be 1273. (But notice that if we instead use 1272 cases per shipment, the difference is only hundredths of a cent)

32. You are the manager of a company that manufactures bicycles, and you buy 6,000 tires a year from a distributor. Each tire costs \$21, the ordering fee is \$20 per shipment, and the storage cost is 96 cents per tire per year. Suppose that the tires are used at a constant rate throughout the year and that each shipment arrives just as the preceding shipment is being used up. How many tires should you order each time to minimize total cost?

Let x = number of tires ordered per shipment.

Want to minimize total cost.

Total cost = (cost of tires) + (cost of ordering) + (cost of storage)

Let y = number of shipments per year.

Must buy 6000 tires per year.

So, (# of tires per shipment) • (# of shipments per year) = 6000.

That is, $xy = 6000$ so $y = \frac{6000}{x}$ or $6000x^{-1}$.

Cost of tires = (6000 tires) • (21 dollars per tire)

$= 6000 \cdot 21$ which is CONSTANT. $(6000 \cdot 21 = 126{,}000)$

Cost of ordering = (20 dollars per shipment) • (# of shipments) = $20y$

Cost of storage = (0.96 dollars per tire) • (# of tires stored) = $0.96x$

Total cost $= 126{,}000 + 20y + 0.96x$

$= 126{,}000 + 20 \cdot 6000x^{-1} + 0.96x$

THIS IS WHAT WE WANT TO MINIMIZE

$C = 126{,}000 + 120{,}000x^{-1} + 0.96x$

$C'(x) = -120{,}000x^{-2} + 0.96$

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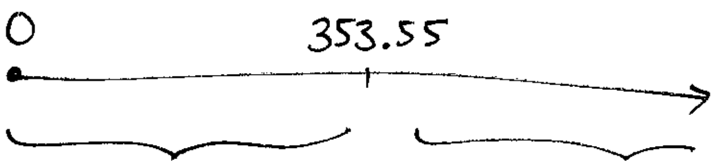
Critical numbers? $\dfrac{-120{,}000}{x^2} + 0.96 = 0$

$$0.96 = \frac{120{,}000}{x^2}$$

$$0.96x^2 = 120{,}000$$

$$x^2 = \frac{120{,}000}{0.96}$$

$$\Rightarrow x^2 = 125{,}000 \Rightarrow x = \sqrt{125{,}000} = 353.55$$



If $x = 1$ then $C' = \dfrac{-120{,}000}{x^2} + 0.96$ is negative

C is decreasing

If $x = 1{,}000{,}000$ then $C' = \dfrac{-120{,}000}{x^2} + 0.96$ is positive

C is increasing

So C is minimized if $x = 353.55$.

Since x must be a whole number, check $x = 353$, $x = 354$

$$C(353) = 126{,}000 + \frac{120{,}000}{353} + 0.96 \cdot 353 = 126{,}678.8233$$

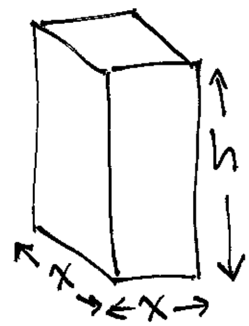
$$C(354) = 126{,}000 + \frac{120{,}000}{354} + 0.96 \cdot 354 = 126{,}678.8231$$

Cost is minimized if we order 354 tires each time.

33. A carpenter has been asked to build an open box with a square base. The sides of the box will cost \$3 per square meter, and the base will cost \$4 per square meter. What are the dimensions of the box of greatest volume that can be constructed for \$48?

DRAW A PICTURE and MAKE UP VARIABLE NAMES.

Open box with square base $\rightarrow$ let's say base is x by x square and height is h.



Four walls, each of which is an x by h rectangle

Bottom is an x by x square

Want to MAXIMIZE the volume. Volume $= x^2 h$

Total cost must be 48 dollars.

Total cost $=$ (cost of four walls) $+$ (cost of bottom)

$=$ 4$\cdot$(area of wall)$\cdot$(3 dollars per sq m) $+$ (area of bottom)$\cdot$(4 \$ per sq m)

$= 4 \cdot xh \cdot 3 + x^2 \cdot 4 \quad = 12xh + 4x^2.$

Total cost $= 48$, so $12xh + 4x^2 = 48$

$$\Rightarrow \quad 3xh + x^2 = 12$$

$$3xh = 12 - x^2$$

$$h = \frac{12 - x^2}{3x}$$

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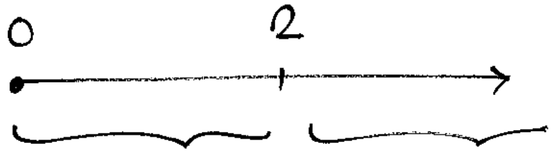
$$\text{Volume} = x^2 h = x^2 \cdot \frac{12 - x^2}{3x} = x \cdot \frac{12 - x^2}{3}$$

THIS IS WHAT WE WANT TO MAXIMIZE

$$V = \frac{12x - x^3}{3} = 4x - \frac{x^3}{3}$$

$V'(x) = 4 - x^2$ Critical numbers? $V' = 0$?

$x = 2$



If $x = 1$	If $x = 10$ (say)
then $V' = 4 - x^2$	then $V' = 4 - x^2$
is positive	is negative
V is increasing	V is decreasing

So V is maximized if $x = 2$.

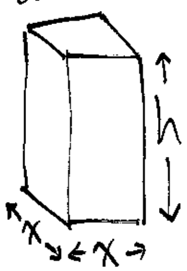
If $x = 2$ then $h = \frac{12 - x^2}{3x} = \frac{12 - 4}{6} = \frac{8}{6} = \frac{4}{3}$

The base of the box should be 2 meters by 2 meters and the height should be $\frac{4}{3}$ meters.

34. Carla is a carpenter who has been hired to make a closed box with a square base and a volume of 250 cubic meters. The material for the top and bottom of the box costs \$2 per square meter, and the material for the sides costs \$1 per square meter. Can Carla construct the box for less than \$300?

Closed box has top and bottom.

Square base. Say the base is x meters by x meters and the height is h meters.



$$\text{Volume} = x^2 h = 250$$

$$\text{Therefore } h = \frac{250}{x^2}$$

To answer the question, check if MINIMUM cost is less than \$300.

Total cost = (cost of four walls) + (cost of top and bottom)

$= 4 \cdot (\text{area of wall}) \cdot (1 \text{ dollar per sq m}) + 2 \cdot (\text{area of top or bottom}) \cdot (2 \text{ dollars per sq m})$

$= 4 \cdot xh \cdot 1 + 2 \cdot x^2 \cdot 2 = 4xh + 4x^2$ THIS IS WHAT WE WANT TO MINIMIZE

$$C = 4xh + 4x^2 = 4x \cdot \frac{250}{x^2} + 4x^2 = 1000x^{-1} + 4x^2$$

$C'(x) = -1000x^{-2} + 8x$. Crit. numbers? $C' = 0$?

$$\frac{-1000}{x^2} + 8x = 0$$

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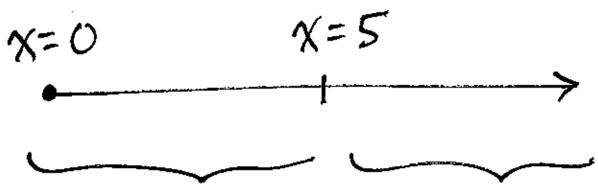
$$8x = \frac{1000}{x^2}$$

$$8x^3 = 1000$$

$$x^3 = \frac{1000}{8} \Rightarrow x = \frac{10}{2} = 5$$

If $x = 5$ then $C = \frac{1000}{x} + 4x^2 = \frac{1000}{5} + 4 \cdot 25$

$= 200 + 100 = 300.$



If $x=1$ then $C' = \frac{-1000}{x^2} + 8x$ is negative

C is decreasing

If $x = 100$ (say) then $C' = \frac{-1000}{x^2} + 8x$ is positive

C is increasing

So cost C is MINIMIZED when $x = 5$.

When $x = 5$, the cost is 300 dollars.

So no, Carla cannot construct the box for LESS than 300 dollars.