

MAC2233

Suggested problems on Chapter 4 material (exponential and logarithmic functions)

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1. Suppose \$1,000 is invested at an annual interest rate of 7%. Compute the future value of the investment after 10 years if the interest is compounded:
 - a. Annually
 - b. Quarterly
 - c. Monthly
 - d. Continuously

2. Suppose \$5,000 is invested at an annual interest rate of 10%. Compute the future value of the investment after 10 years if the interest is compounded:
- a. Annually
 - b. Semiannually
 - c. Daily (using 365 days per year)
 - d. Continuously

- 3.** Find the present value of \$10,000 over a term of 5 years at an annual interest rate of 7% if interest is compounded:
- a.** Annually
 - b.** Quarterly
 - c.** Daily (using 365 days per year)
 - d.** Continuously

4. Find the present value of \$25,000 over a term of 10 years at an annual interest rate of 5% if interest is compounded:
- a. Semiannually
 - b. Monthly
 - c. Continuously

5. Find the effective interest rate r_e for an annual interest rate of 6% that is compounded quarterly.

6. Find the effective interest rate r_e for an annual interest rate of 8% that is compounded daily (use $k = 365$).

7. Esmeralda needs \$5,000 for a trip to Peru when she graduates from college in 4 years. How much must she invest now at an annual interest rate of 5% compounded continuously to achieve her goal?

8. Lyle buys a rare stamp for \$500. If the annual rate of inflation is 4%, how much should he ask when he sells it in 5 years to break even?

9. How quickly will money double if it is invested at an annual interest rate of 7% compounded continuously?

10. Money deposited in a certain bank doubles every 13 years. The bank compounds interest continuously. What annual interest rate does the bank offer?

11. How long will it take for a quantity of money A_0 to triple in value if it is invested at an annual interest rate r compounded continuously?

12. If an account that earns interest compounded continuously takes 12 years to double in value, how long will it take to triple in value?

- 13.** The Morenos invest \$10,000 in an account that grows to \$12,000 in 5 years. What is the annual interest rate r if interest is compounded
- Quarterly
 - Continuously

14. An economist has compiled this data on the gross domestic product (GDP) of a certain country:

Year	1995	2005
GDP (in billions)	100	180

Use this data to predict the GDP in the year 2015 if the GDP is growing:

- Linearly, so that $\text{GDP} = at + b$.
- Exponentially, so that $\text{GDP} = Ae^{kt}$.

15. Tests of an artifact discovered at the Debert site in Nova Scotia show that 28% of the original ^{14}C is still present. Approximately how old is the artifact? (The half-life of ^{14}C is 5,730 years.)

16. A medical student studying the growth of bacteria in a certain culture has compiled this data:

Number of minutes	0	20
Number of bacteria	6,000	9,000

Use this data to find an exponential function of the form $Q(t) = Q_0e^{kt}$ expressing the number of bacteria in the culture as a function of time. How many bacteria are present after 1 hour?

17. The half-life of radium is 1,690 years. How long will it take for a 50-gram sample of radium to be reduced to 5 grams?

18. Differentiate the function.

$$f(x) = e^{x^2+2x-1}$$

19. Differentiate the function.

$$f(x) = (x^2 + 3x + 5)e^{6x}$$

20. Differentiate the function.

$$f(x) = xe^{-x^2}$$

21. Differentiate the function.

$$f(x) = (1 - 3e^x)^2$$

22. Differentiate the function.

$$f(x) = \sqrt{1 + e^x}$$

23. Differentiate the function.

$$f(x) = e^{\sqrt{3x}}$$

24. Differentiate the function.

$$f(x) = e^{1/x}$$

25. Differentiate the function.

$$f(x) = e^x \ln x$$

26. Differentiate the function.

$$F(x) = \ln(2x^3 - 5x + 1)$$

27. Differentiate the function.

$$f(x) = \ln(e^{-x} + x)$$

28. Differentiate the function.

$$g(u) = \ln(u + \sqrt{u^2 + 1})$$

29. Differentiate the function.

$$L(x) = \ln\left(\frac{x^2 + 2x - 3}{x^2 + 2x + 1}\right)$$

30. Find the **second** derivative of the function.

$$f(x) = e^{2x} + 2e^{-x}$$

31. Find the **second** derivative of the function.

$$f(x) = \ln(2x) + x^2$$

32. Use logarithmic differentiation to find the derivative $f'(x)$.

$$f(x) = \frac{(x+2)^5}{(3x-5)^{1/6}}$$

33. Use logarithmic differentiation to find the derivative $f'(x)$.

$$f(x) = \left(\frac{2x+1}{1-3x}\right)^{1/4}$$

34. Use logarithmic differentiation to find the derivative $f'(x)$.

$$f(x) = (x + 1)^3(6 - x)^2(2x + 1)^{1/3}$$

35. Use logarithmic differentiation to find the derivative $f'(x)$.

$$f(x) = \frac{e^{-3x}\sqrt{2x-5}}{(6-5x)^4}$$

36. Use logarithmic differentiation to find the derivative $f'(x)$.

$$f(x) = 5^{x^2}$$

- 37.** A certain industrial machine depreciates so that its value after t years becomes $Q(t) = 20,000e^{-0.4t}$ dollars.
- At what rate is the value of the machine changing with respect to time after 5 years?
 - At what percentage rate is the value of the machine changing with respect to time after t years? Does this percentage rate depend on t or is it constant?

38. The total number of hamburgers sold by a national fast-food chain is growing exponentially. If 4 billion had been sold by 2005 and 12 billion had been sold by 2010, how many will have been sold by 2015?

39. Once the initial publicity surrounding the release of a new book is over, sales of the hardcover edition tend to decrease exponentially. At the time publicity was discontinued, a certain book was experiencing sales of 25,000 copies per month. One month later, sales of the book had dropped to 10,000 copies per month. What will the sales be after one more month?

40. It is estimated that the population of a certain country grows exponentially. If the population was 60 million in 1997 and 90 million in 2002, what will the population be in 2012?