WRITE YOUR NAME:

MAC 2233 Homework 1
Due in class, Friday January 19th
You can use more paper if necessary, but please STAPLE

Question 1. Factor the following completely.

$$2y^6 - 32y^2$$

$$= 2y^2(y^4 - 16)$$

$$= 2y^2(y^2+4)(y^2-4)$$

$$= 2y^2(y^2+4)(y+2)(y-2)$$

Question 2. If $f(x) = x^2$, evaluate and simplify each of the following.

- $f(1.3)$ $= (1.3)^2 = 1.69$
- $f(1+h)$ $= (1+h)^2 = 1 + 2h + h^2$
- $f(1.003)$ $= (1.003)^2 = 1.006009$
- $f(3.001) - f(3)$

$$= (3.001)^2 - 3^2$$
$$= 9.006001 - 9 = 0.006001$$

Notice that $(1.003)^2 = (1 + 0.003)^2$
which is of the form $(1+h)^2$.

$$(1.003)^2 = \underbrace{(1+0.003)^2}_{(1+h)^2} = \underbrace{1 + 2(0.003) + (0.003)^2}_{1+2h+h^2}$$
$$= 1 + 0.006 + 0.000\,009$$

Similarly, $(3.001)^2$ is of the form $(3+h)^2$.

$$(3+h)^2 = 3^2 + 2\cdot 3\cdot h + h^2 = 9 + 6h + h^2$$
$$(3+0.001)^2 = 9 + 6(0.001) + (0.001)^2$$
$$= 9 + 0.006 + 0.000\,001$$

If you're comfortable with algebra, then $(1+h)^2$ and $(3+h)^2$ are <u>easier</u> than $(1.003)^2$ and $(3.001)^2$.
(and more general)

Question 3. Evaluate the limit.

$$\lim_{x\to 9} \frac{3-\sqrt{x}}{9-x}$$

$$\lim_{x\to 9} \frac{3-\sqrt{x}}{9-x} \cdot \frac{3+\sqrt{x}}{3+\sqrt{x}}$$

$$= \lim_{x\to 9} \frac{3^2-(\sqrt{x})^2}{(9-x)(3+\sqrt{x})}$$

$$= \lim_{x\to 9} \frac{9-x}{(9-x)(3+\sqrt{x})}$$

$$= \lim_{x\to 9} \frac{1}{3+\sqrt{x}}$$

$$= \frac{1}{3+\sqrt{9}} = \frac{1}{3+3} = \frac{1}{6}$$

Question 4. Evaluate the limit.

$$\lim_{x\to\infty} \frac{x+3}{x^2+9} = 0$$

Algebraic method:

$$\lim_{x\to\infty} \frac{x+3}{x^2+9} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}}$$

$$= \lim_{x\to\infty} \frac{\frac{x}{x^2}+\frac{3}{x^2}}{\frac{x^2}{x^2}+\frac{9}{x^2}} = \lim_{x\to\infty} \frac{\frac{1}{x}+\frac{3}{x^2}}{1+\frac{9}{x^2}}$$

$$= \frac{0+0}{1+0} = \frac{0}{1} = 0.$$

Also allowed:

$$\lim_{x\to\infty} \frac{x+(\text{smaller powers of } x)}{x^2+(\text{smaller powers of } x)} = \lim_{x\to\infty} \frac{x}{x^2}$$

$$= \lim_{x\to\infty} \frac{1}{x} = 0$$

Also allowed to use the following shortcut:
x is extreme, and bottom has higher degree than top,
so answer is 0

Question 5. Evaluate the limit.

$$\lim_{x\to\infty} \frac{6x^2+5}{3x^2+\sqrt{x}} = \frac{6}{3} = 2$$

Algebraic method: $\displaystyle \lim_{x\to\infty} \frac{6x^2+5}{3x^2+\sqrt{x}} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}}$

$$= \lim_{x\to\infty} \frac{\frac{6x^2}{x^2}+\frac{5}{x^2}}{\frac{3x^2}{x^2}+\frac{x^{1/2}}{x^2}} = \lim_{x\to\infty} \frac{6+\frac{5}{x^2}}{3+\frac{1}{x^{3/2}}}$$

$$= \frac{6+0}{3+0} = \frac{6}{3} = 2$$

Also allowed: $\displaystyle \lim_{x\to\infty} \frac{6x^2 + (\text{smaller powers of } x)}{3x^2 + (\text{smaller powers of } x)}$

$$= \lim_{x\to\infty} \frac{6x^2}{3x^2} \quad (\text{since } x \text{ is extreme})$$

$$= \lim_{x\to\infty} \frac{6}{3} = \frac{6}{3} = 2.$$