WRITE YOUR NAME:

MAC 2233 Homework 2
Due in class, Friday February 2nd
You can use more paper if necessary, but please STAPLE

Question 1. Consider the function $f(x) = 3x^2$.

(a) If x changes from 10 to 10.1, how much does $f(x)$ change by?

(b) Let P be the point on the graph of f where $x = 10$, and let Q be the point on the graph of f where $x = 10.1$. What's the slope of the line joining P and Q?

(c) Let P be the point on the graph of f where $x = 10$, and let R be the point on the graph of f where $x = 10 + h$. What's the slope of the line joining P and R?

(d) What's the slope of the tangent line to the graph of f at the point P?

$$\begin{array}{r} 10.1 \\ \times 10.1 \\ \hline 101 \\ 101 \\ \hline 102.01 \end{array}$$

Also $(10.1)^2 = (10+0.1)^2 = 10^2 + 2\cdot 10\cdot 0.1 + 0.1^2 = 100 + 2 + 0.01$

(a) $f(10.1) - f(10) = 3(10.1)^2 - 3\cdot(10)^2$
$= 3\cdot(102.01) - 3\cdot(100) = 306.03 - 300 = 6.03.$

(b) At the point P, we have $x = 10$ and $y = 3\cdot(10)^2 = 300$.
At the point Q, we have $x = 10.1$ and $y = 3\cdot(10.1)^2 = 306.03$.

Slope of line $= \dfrac{\text{change in } y}{\text{change in } x} = \dfrac{306.03 - 300}{10.1 - 10} = \dfrac{6.03}{0.1} = 60.3$

(c) At the point P, we have $x = 10$ and $y = 300$ like in part (b).
At the point R, we have $x = 10 + h$ and $y = 3(10+h)^2$
$= 3(100 + 20h + h^2) = 300 + 60h + 3h^2$.

Slope of line $= \dfrac{\text{change in } y}{\text{change in } x} = \dfrac{(300 + 60h + 3h^2) - 300}{(10+h) - 10} = \dfrac{60h + 3h^2}{h}$

$= \dfrac{(60+3h)h}{h} = 60 + 3h.$

(d) Slope of tangent line at point P is $f'(x) = 6x$ evaluated at $x = 10$ which is $6\cdot 10 = 60$. (This is also limit of previous answer as $h \to 0$.)

Question 2. Find the derivative of the function.

$$f(x) = \frac{x^5}{5} + \frac{5}{x^5} + \frac{1}{5x^5} + \frac{x^{47} + x^{48}}{x^5}$$

$$f(x) = \frac{1}{5}x^5 + 5x^{-5} + \frac{1}{5}x^{-5} + \frac{x^{47}}{x^5} + \frac{x^{48}}{x^5}$$

$$f(x) = \frac{1}{5}x^5 + 5x^{-5} + \frac{1}{5}x^{-5} + x^{42} + x^{43}$$

$$f'(x) = \frac{1}{5}\cdot 5x^4 + 5(-5)x^{-6} + \frac{1}{5}(-5)x^{-6} + 42x^{41} + 43x^{42}$$

This can be simplified a little

$$f'(x) = x^4 - 25x^{-6} - x^{-6} + 42x^{41} + 43x^{42}$$

$$= x^4 - 26x^{-6} + 42x^{41} + 43x^{42}$$

$$\text{or } x^4 - \frac{26}{x^6} + 42x^{41} + 43x^{42}$$

$$\text{or even } \frac{x^{10}}{x^6} - \frac{26}{x^6} + \frac{42x^{47}}{x^6} + \frac{43x^{48}}{x^6} = \frac{43x^{48} + 42x^{47} + x^{10} - 26}{x^6}$$