WRITE YOUR NAME:

MAC 2233 Homework 3
Due in class, Friday February 9th
You can use more paper if necessary, but please STAPLE

Question 1. Differentiate the given function.

$$f(x) = \frac{x^7 - 8x^5}{x^3}$$

Easiest way:

$$f(x) = \frac{x^7 - 8x^5}{x^3} = \frac{x^7}{x^3} - \frac{8x^5}{x^3} = x^4 - 8x^2$$

so $f'(x) = 4x^3 - 8 \cdot 2x$

$$= 4x^3 - 16x$$

Question 2. Find the **equation** of the line that is tangent to the graph of the given function at the point where $x = 0$.

$$y = (x^2 + 5x + 4)(x^2 + 5x + 6)$$

If $x=0$ then $y = (0^2 + 5\cdot 0 + 4)(0^2 + 5\cdot 0 + 6)$

$= (0+0+4)(0+0+6) = 4\cdot 6 = 24$

The point of tangency is $(0,24)$ $\begin{matrix} x=0 \\ y=24 \end{matrix}$

Slope? $\frac{dy}{dx} = y' = (x^2+5x+4)'(x^2+5x+6) + (x^2+5x+4)(x^2+5x+6)'$

$= (2x+5)(x^2+5x+6) + (x^2+5x+4)(2x+5)$

If $x=0$

then $y' = (2\cdot 0+5)(0^2+5\cdot 0+6) + (0^2+5\cdot 0+4)(2\cdot 0+5)$

$= (0+5)(0+0+6) + (0+0+4)(0+5)$

$= 5\cdot 6 + 4\cdot 5 = 30+20 = 50$

Slope is 50 and point is $(0,24)$

Equation of line is $y - 24 = 50(x-0)$

or $y - 24 = 50x$

or $y = 50x + 24$

Question 3. Differentiate the function and simplify your answer.

$$f(x) = \frac{1}{\sqrt{3x^2 - 1}}$$

Easiest way: $f(x) = \dfrac{1}{(3x^2-1)^{1/2}} = (3x^2-1)^{-1/2}$

So by chain rule,

$$f'(x) = \underbrace{-\frac{1}{2}(3x^2-1)^{-3/2}}_{\substack{\text{Took deriv. of outside} \\ \text{Left inside alone}}} \cdot \underbrace{(3x^2-1)'}_{\text{Deriv. of inside}}$$

$$= -\frac{1}{2}(3x^2-1)^{-3/2} \cdot 6x$$

$$= -3x(3x^2-1)^{-3/2} \quad \text{or} \quad \frac{-3x}{(3x^2-1)^{3/2}}$$

Alternative method in case it helps:

$f = (3x^2-1)^{-1/2} \Rightarrow f = u^{-1/2}$ and $u = 3x^2 - 1$

$$\frac{df}{du} = -\frac{1}{2}u^{-3/2} \qquad \frac{du}{dx} = 6x$$

$$\frac{df}{dx} = \frac{df}{du} \cdot \frac{du}{dx} = -\frac{1}{2}u^{-3/2} \cdot 6x = -\frac{1}{2}(3x^2-1)^{-3/2} \cdot 6x$$

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$$

Question 4. Differentiate the function and simplify your answer.

$$f(x) = \frac{(x+1)^3}{(x-1)^8}$$

$$f'(x) = \frac{[(x+1)^3]'(x-1)^8 - (x+1)^3[(x-1)^8]'}{((x-1)^8)^2}$$

$$= \frac{3(x+1)^2 \cdot 1 \cdot (x-1)^8 - (x+1)^3 \cdot 8(x-1)^7 \cdot 1}{(x-1)^{16}}$$

Next, can take out common factor

$$= \frac{(x+1)^2(x-1)^7\left(3 \cdot 1 \cdot (x-1) - (x+1) \cdot 8 \cdot 1\right)}{(x-1)^{16}}$$

$$= \frac{(x+1)^2(x-1)^7\left(3x - 3 - (8x+8)\right)}{(x-1)^{16}} = \frac{(x+1)^2(x-1)^7(3x-3-8x-8)}{(x-1)^{16}}$$

$$= \frac{(x+1)^2(x-1)^7(-5x-11)}{(x-1)^{16}} = \frac{(x+1)^2(-5x-11)}{(x-1)^9}$$

or $\dfrac{-(x+1)^2(5x+11)}{(x-1)^9}$

Question 5. Consider the function $f(x) = 6x^2 - x^3$.

(a) Find $f'(x)$ and $f''(x)$.

(b) Evaluate $f(x)$ and $f'(x)$ at $x = 0$, 1, 2, 3, 4, 5, and 6.

(c) Use the information from (b) to draw a graph of $f(x)$ on the interval $0 \le x \le 6$.

(d) What value of x do you think makes $f(x)$ the largest? What value of x do you think makes $f'(x)$ the largest? What does that say about the graph at those points?

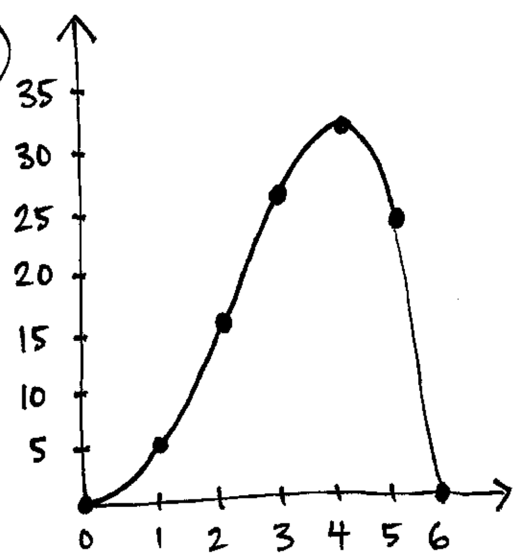
(a) $f(x) = 6x^2 - x^3 \Rightarrow f'(x) = 6 \cdot 2x - 3x^2 = 12x - 3x^2$

$\Rightarrow f''(x) = 12 - 3 \cdot 2x = 12 - 6x.$

(b)

x	$f(x) = 6x^2 - x^3 = x^2 \cdot (6-x)$	$f'(x) = 12x - 3x^2 = 3x \cdot (4-x)$
0	$0^2 \cdot 6 = 0$	$0 \cdot 4 = 0$
1	$1^2 \cdot 5 = 5$	$3 \cdot 3 = 9$
2	$2^2 \cdot 4 = 4 \cdot 4 = 16$	$6 \cdot 2 = 12$
3	$3^2 \cdot 3 = 9 \cdot 3 = 27$	$9 \cdot 1 = 9$
4	$4^2 \cdot 2 = 16 \cdot 2 = 32$	$12 \cdot 0 = 0$
5	$5^2 \cdot 1 = 25 \cdot 1 = 25$	$15 \cdot (-1) = -15$
6	$6^2 \cdot 0 = 0$	$18 \cdot (-2) = -36$

(c)



(d) It appears that $x = 4$ makes $f(x)$ the largest. (But what if $x = 3.9$ or 4.1?)

It appears that $x = 2$ makes $f'(x)$ the largest. (But what if $x = 1.9$ or 2.1?)

$f(x)$ large as possible: highest point on graph

$f'(x)$ large as possible: steepest point on graph