

WRITE YOUR NAME:

MAC 2233 Homework 4

Due in class, Friday February 23rd

You can use more paper if necessary, but please STAPLE

Question 1. Let $f(x) = \frac{x^2}{x-2}$. Find the intervals where $f(x)$ is increasing, decreasing, concave up, and concave down.

Use this information to draw a reasonable graph of the function f on the interval $-10 \leq x \leq 10$.

$$f'(x) = \frac{(x^2)'(x-2) - x^2(x-2)'}{(x-2)^2} = \frac{2x(x-2) - x^2 \cdot 1}{(x-2)^2}$$

$$= \frac{2x^2 - 4x - x^2}{(x-2)^2} = \frac{x^2 - 4x}{(x-2)^2} \quad \text{or} \quad \frac{x(x-4)}{(x-2)^2}$$

$$f''(x) = \frac{(x^2 - 4x)'(x-2)^2 - (x^2 - 4x)[(x-2)^2]'}{((x-2)^2)^2}$$

$$= \frac{(2x-4)(x-2)^2 - (x^2-4x) \cdot 2(x-2) \cdot 1}{(x-2)^4}$$

*deriv. of $x-2$
(chain rule)*

$$= \frac{(x-2) \left((2x-4)(x-2) - (x^2-4x) \cdot 2 \right)}{(x-2)^4}$$

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f is increasing on $(-\infty, 0)$
decreasing on $(0, 2)$
decreasing on $(2, 4)$
increasing on $(4, \infty)$

f is concave down on $(-\infty, 2)$ and concave up on $(2, \infty)$

Reasonable graph? Note that $f(x) = \frac{x^2}{x-2}$

is undefined if $x=2$. If $x \rightarrow 2^+$, then $\frac{x^2}{x-2} = \frac{\text{near } 4}{\text{positive near } 0}$

Vertical asymptote at $x=2$. If $x \rightarrow 2^-$, then $\frac{x^2}{x-2} = \frac{\text{near } 4}{\text{negative near } 0}$

Question asks for graph on interval $[-10, 10]$.

$$f(-10) = \frac{(-10)^2}{-10-2} = \frac{100}{-12} \approx -8$$

$$f(10) = \frac{10^2}{10-2} = \frac{100}{8} \approx 12$$

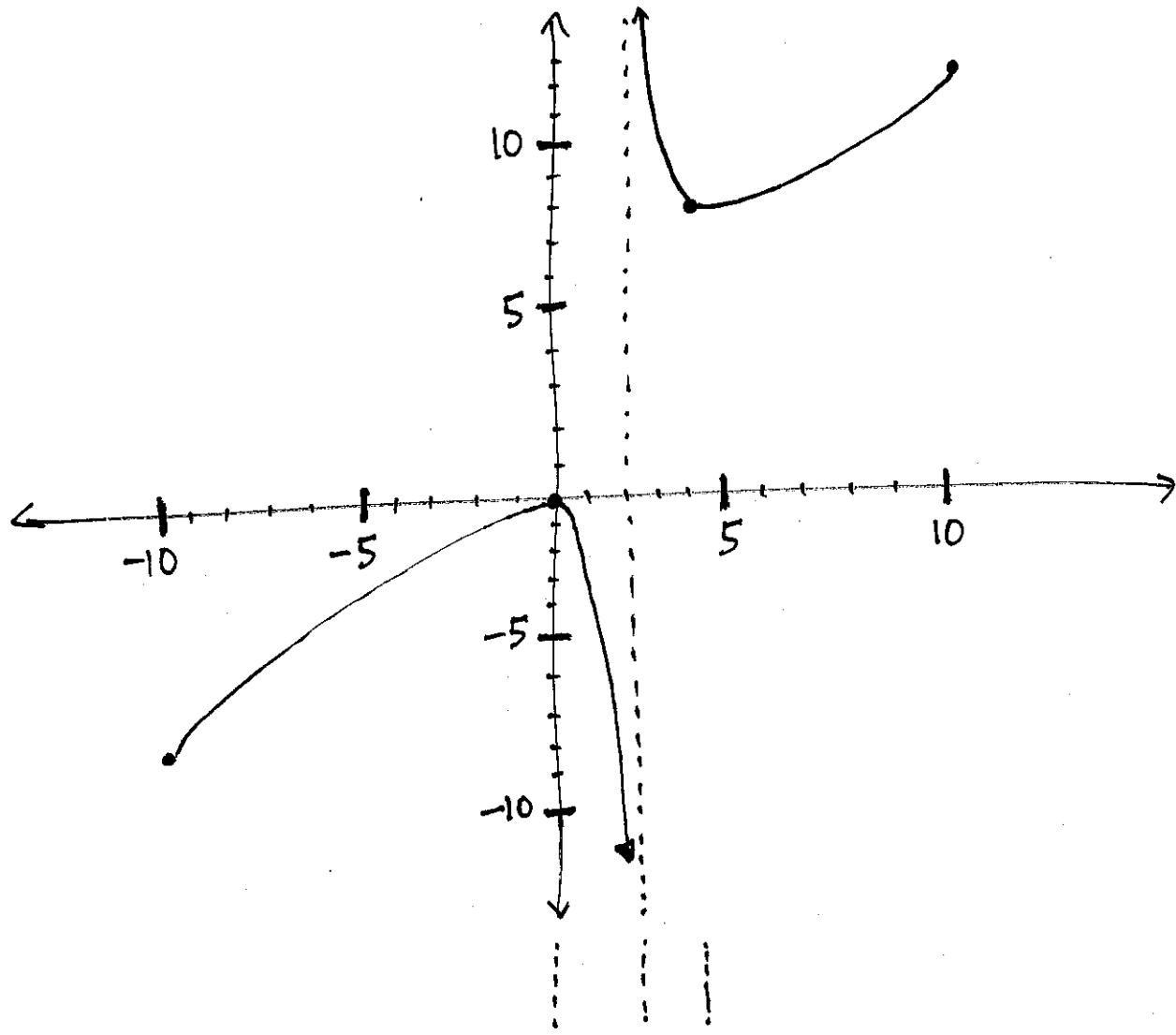
Other x values that are "special": $x=0$, $x=4$
(f switches between incr. and dec.)

$$f(0) = \frac{0^2}{0-2} = \frac{0}{-2} = 0$$

$$f(4) = \frac{4^2}{4-2} = \frac{16}{2} = 8$$

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Reasonable hand-drawn graph on interval $[-10, 10]$:



f incr and conc down
if $x < 0$

f decreasing
and conc down
if $0 < x < 2$

f incr and conc up
if $x > 4$

f decreasing
and conc up
if $2 < x < 4$