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MAC 2233 Section U02 Test 1
Friday January 26th
Total possible score: 30 points (5 points per page)

[5 points] Question 1. Evaluate the limit.

$$\lim_{x \to 5} \frac{x-5}{x^2-25}$$

$$\lim_{x \to 5} \frac{x-5}{(x+5)(x-5)}$$

$$= \lim_{x \to 5} \frac{1}{x+5}$$

$$= \frac{1}{5+5} = \frac{1}{10}$$

[2 points] **Question 2a.** Evaluate the limit.

$$\lim_{x\to\infty} \frac{4x^9 + x^2 + 1}{7x^9 - x^3 + 5} = \frac{4}{7}$$

Algebraic method: $\displaystyle \lim_{x\to\infty} \frac{4x^9 + x^2 + 1}{7x^9 - x^3 + 5} \cdot \frac{\frac{1}{x^9}}{\frac{1}{x^9}}$

$$= \lim_{x\to\infty} \frac{4 + \frac{1}{x^7} + \frac{1}{x^9}}{7 - \frac{1}{x^6} + \frac{5}{x^9}} = \frac{4+0+0}{7-0+0} = \frac{4}{7}$$

[3 points] **Question 2b.** Evaluate the limit.

$$\lim_{x\to-\infty} \frac{-2x^6 + 1}{3x^7 + 2x^4 - 9} = 0$$

Algebraic method: $\displaystyle \lim_{x\to-\infty} \frac{-2x^6 + 1}{3x^7 + 2x^4 - 9} \cdot \frac{\frac{1}{x^7}}{\frac{1}{x^7}}$

$$= \lim_{x\to-\infty} \frac{-\frac{2}{x} + \frac{1}{x^7}}{3 + \frac{2}{x^3} - \frac{9}{x^7}} = \frac{0+0}{3+0-0}$$

$$= \frac{0}{3} = 0$$

[3 points] **Question 3a.** Find $\lim_{x\to 2^-} f(x)$ and $\lim_{x\to 2^+} f(x)$, where $f(x)$ is as defined below.

$$f(x) = \begin{cases} 3x^2 - 5 & \text{if } x < 2 \\ 3 + 2x & \text{if } x \geq 2 \end{cases}$$

$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} (3x^2 - 5)$$

$$= 3\cdot 2^2 - 5 = 3\cdot 4 - 5$$

$$= 12 - 5 = 7$$

$$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} (3 + 2x)$$

$$= 3 + 2\cdot 2 = 3 + 4 = 7$$

[2 points] **Question 3b.** List all values of x for which the function is not continuous.

$$f(x) = \frac{x-5}{x^2 - 5x}$$

$$f(x) = \frac{x-5}{x(x-5)}$$

f is discontinuous whenever denominator is 0

f is discontinuous at $x = 0$ and at $x = 5$.

[5 points] **Question 4.** Find the value of the constant A that makes the given function continuous for all x.

$$f(x) = \begin{cases} \dfrac{x^2-1}{x+1} & \text{if } x < -1 \\ Ax^2 + x - 3 & \text{if } x \geq -1 \end{cases}$$

To make f continuous at $x = -1$,

we need $\lim\limits_{x \to -1^-} f(x) = \lim\limits_{x \to -1^+} f(x)$.

$$\lim_{x \to -1^-} f(x) = \lim_{x \to -1^-} \frac{x^2-1}{x+1} = \lim_{x \to -1^-} \frac{(x+1)(x-1)}{(x+1)}$$

$$= \lim_{x \to -1^-} (x-1) = -1-1 = -2$$

$$\lim_{x \to -1^+} f(x) = \lim_{x \to -1^+} (Ax^2 + x - 3)$$

$$= A \cdot (-1)^2 + (-1) - 3$$

$$= A \cdot 1 - 1 - 3 = A - 4$$

So, we must have

$$A - 4 = -2$$

$$A = -2 + 4$$

$$\underline{A = 2}$$

[5 points] **Question 5.** Consider the curve defined by $y = x^2$. Let P be the point on the curve where $x = 5$, and let Q be the point on the curve where $x = 5 + h$. Find the slope of the line joining P and Q.

Curve $y = x^2$

Point P: $x = 5 \Rightarrow y = 5^2 = 25$

Point Q: $x = 5+h \Rightarrow y = (5+h)^2$

$$= 25 + 10h + h^2$$

Slope of line joining P and Q:

$$\text{Slope} = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{(5+h)^2 - 5^2}{(5+h) - 5} = \frac{(25 + 10h + h^2) - 25}{h}$$

$$= \frac{10h + h^2}{h} = \frac{(10+h)h}{h} = \underline{10 + h}$$

(And then <u>if</u> we let h approach 0, that slope approaches 10)

$$f(\square) = \frac{1}{\square}$$

$$f(x+h) = \frac{1}{x+h}$$

[5 points] **Question 6.** Find the derivative of $f(x) = \dfrac{1}{x}$ using the definition of derivative. Also find the equation of the tangent line to the graph at the point where $x = 1$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h} \cdot \frac{x(x+h)}{x(x+h)}$$

$$= \lim_{h \to 0} \frac{\frac{x(x+h)}{x+h} - \frac{x(x+h)}{x}}{hx(x+h)} = \lim_{h \to 0} \frac{x - (x+h)}{hx(x+h)}$$

$$= \lim_{h \to 0} \frac{x - x - h}{hx(x+h)} = \lim_{h \to 0} \frac{-h}{hx(x+h)} = \lim_{h \to 0} \frac{-1}{x(x+h)}$$

$= \dfrac{-1}{x(x+0)} = \dfrac{-1}{x^2}$. So the slope of the tangent line at $x = 1$ will be $-\dfrac{1}{1^2} = -1$.

At the point where $x = 1$, we have $y = \frac{1}{1} = 1$

So equation of tangent line is $y - 1 = -1(x-1)$.

Can also be written $y - 1 = -x + 1$ or $y = -x + 2$ or $y = 2 - x$