MAC2241
Suggested problems on Chapter 1 material
(functions and sequences)

Idris Mercer

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1. If $f(x) = x^2$, simplify each of the expressions. $f(\square) = (\square)^2$

$$\frac{f(3+h)-f(3)}{h}$$

$$\frac{f(x+h)-f(x)}{h}$$

$$\frac{f(3+h)-f(3)}{h} = \frac{(3+h)^2-3^2}{h} = \frac{9+6h+h^2-9}{h}$$

$$= \frac{6h+h^2}{h} = \frac{(6+h)h}{h} = 6+h$$

$$\frac{f(x+h)-f(x)}{h} = \frac{(x+h)^2-x^2}{h} = \frac{x^2+2xh+h^2-x^2}{h}$$

$$= \frac{2xh+h^2}{h} = \frac{(2x+h)h}{h} = 2x+h$$

2. If $f(x) = \frac{1}{x}$, simplify each of the expressions.

$f(\square) = \frac{1}{\square}$

$$\frac{f(3+h) - f(3)}{h}$$

$$\frac{f(x+h) - f(x)}{h}$$

$$\frac{f(3+h)-f(3)}{h} = \frac{\frac{1}{3+h} - \frac{1}{3}}{h} \cdot \frac{3(3+h)}{3(3+h)}$$

$$= \frac{\frac{3(3+h)}{3+h} - \frac{3(3+h)}{3}}{h \cdot 3(3+h)} = \frac{3-(3+h)}{h\cdot 3(3+h)} = \frac{3-3-h}{h\cdot 3(3+h)}$$

$$= \frac{-h}{h\cdot 3(3+h)} = \frac{-1}{3(3+h)}$$

$$\frac{f(x+h)-f(x)}{h} = \frac{\frac{1}{x+h} - \frac{1}{x}}{h} \cdot \frac{x(x+h)}{x(x+h)}$$

$$= \frac{\frac{x(x+h)}{x+h} - \frac{x(x+h)}{x}}{hx(x+h)} = \frac{x-(x+h)}{hx(x+h)} = \frac{x-x-h}{hx(x+h)}$$

$$= \frac{-h}{hx(x+h)} = \frac{-1}{x(x+h)}$$

3. Find the domain of the function.

$$f(x) = \frac{x+4}{x^2-9}$$

Domain is all x <u>except</u> where $x^2 - 9 = 0$

$$(x+3)(x-3) = 0$$

$$x+3 = 0 \quad \text{or} \quad x-3 = 0$$

$$x = -3 \quad \text{or} \quad x = 3$$

Domain is all x <u>except</u> $x = -3$, $x = 3$.

Can also give answer in interval notation:

$$(-\infty, -3) \cup (-3, 3) \cup (3, \infty)$$

4. Find the domain of the function.

$$f(x) = \frac{2x^3-5}{x^2+x-6}$$

Domain is all x <u>except</u> where $x^2 + x - 6 = 0$

$$(x+3)(x-2) = 0$$

$$x+3 = 0 \quad \text{or} \quad x-2 = 0$$

$$x = -3 \quad \text{or} \quad x = 2$$

Domain is all x <u>except</u> $x = -3$, $x = 2$.

Can also give answer in interval notation:

$$(-\infty, -3) \cup (-3, 2) \cup (2, \infty)$$

5. Find the domain of the function.

$$\sqrt{2-\sqrt{x}}$$

Since formula contains $\sqrt{x}$, we must have $x \geq 0$.
Since formula contains $\sqrt{2-\sqrt{x}}$, we must have $2-\sqrt{x} \geq 0$.

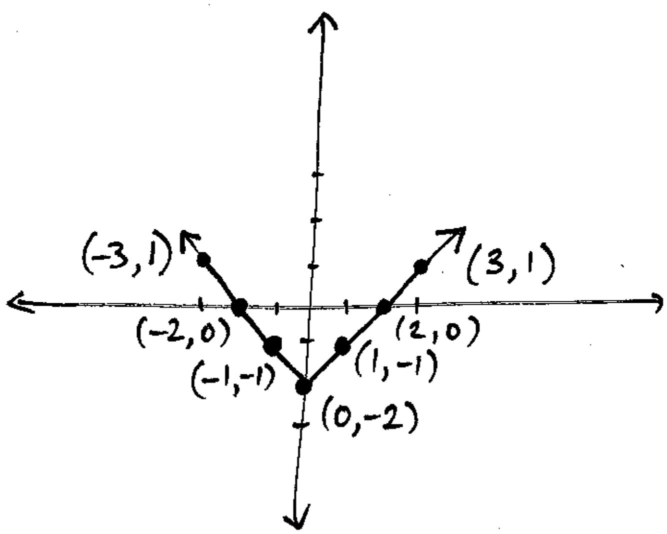
$$2-\sqrt{x} \geq 0$$
$$2 \geq \sqrt{x} \Rightarrow \sqrt{x} \leq 2$$
$$x \leq 4$$

Combining the conditions $x \geq 0$ and $x \leq 4$

we conclude: $\left.\begin{array}{l} 0 \leq x \leq 4. \\ \text{or } [0, 4] \end{array}\right\}$ Domain

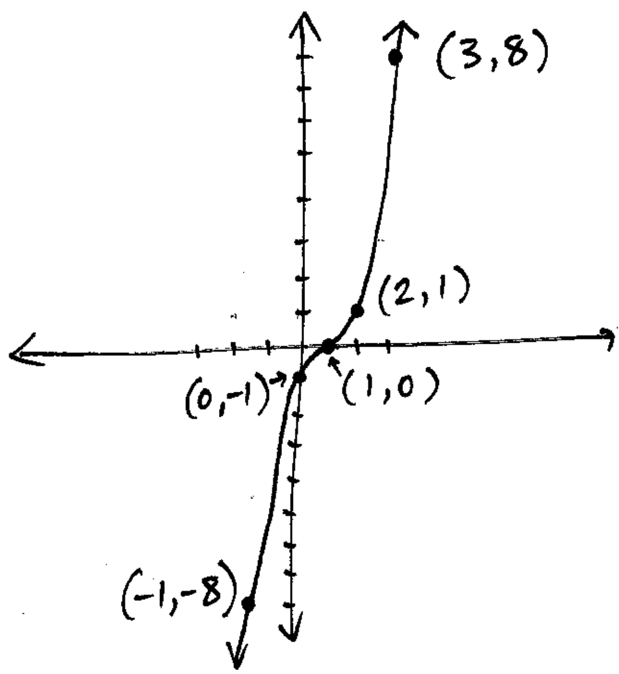
6. Draw a reasonable graph of the function. Label at least three points.

$$f(x) = |x| - 2$$



7. Draw a reasonable graph of the function. Label at least three points.

$$f(x) = (x - 1)^3$$

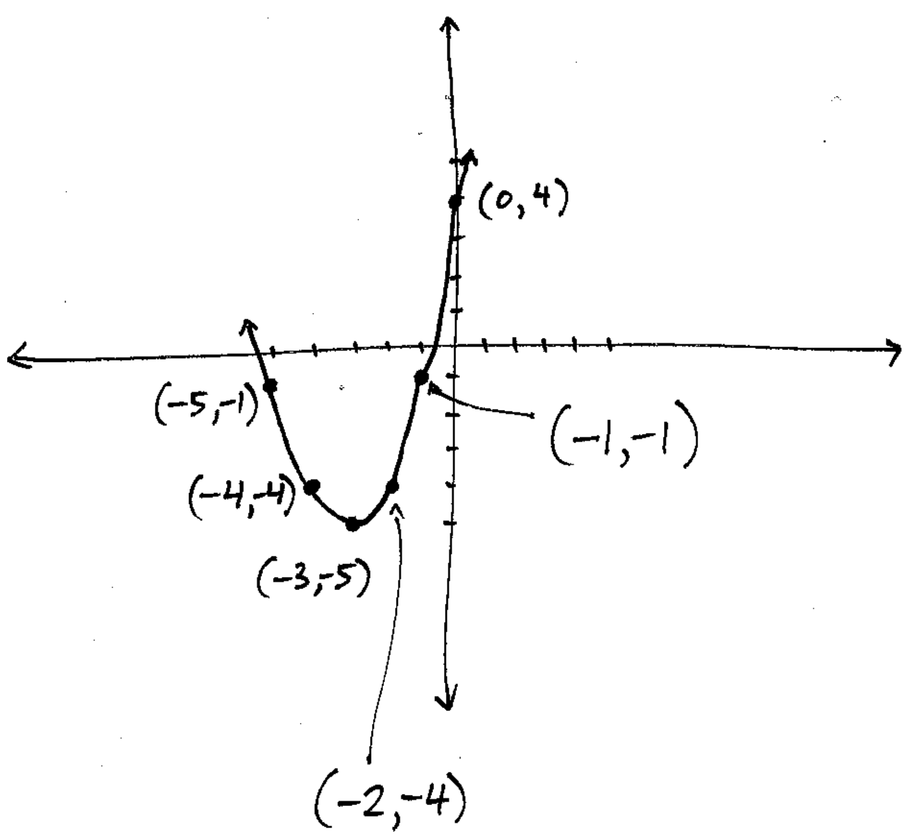


8. Draw a reasonable graph of the function. Label at least three points.

$$f(x) = x^2 + 6x + 4$$

It may help to "complete the square"

$$x^2 + 6x + 4 = x^2 + 6x + 9 - 5 = (x+3)^2 - 5$$

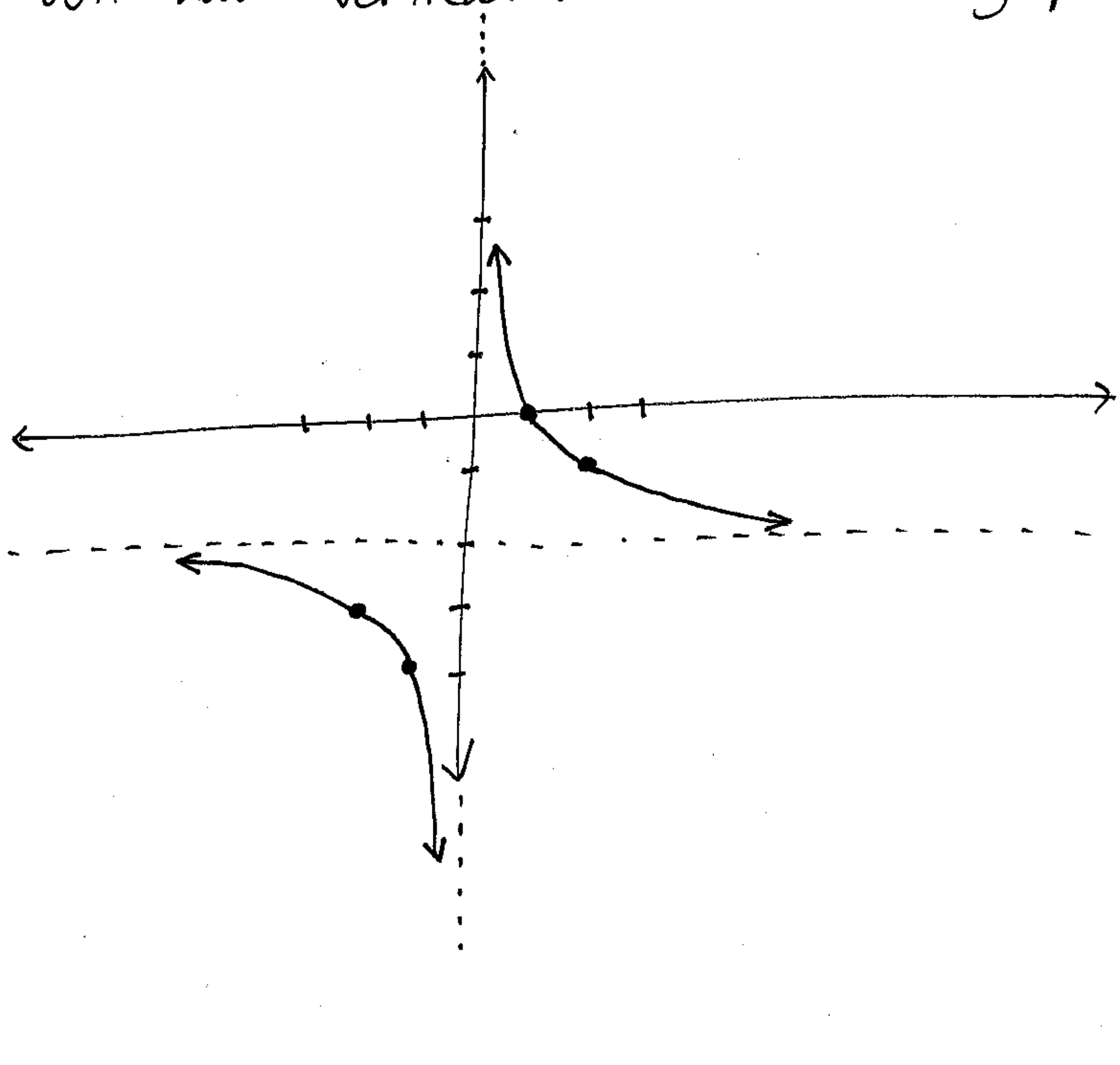


9. Draw a reasonable graph of the function. Label at least three points.

$$f(x) = \frac{2}{x} - 2$$

Resembles graph of $\frac{1}{x}$ but stretched and shifted

Will have vertical and horizontal asymptotes



10. Find $f \circ g$ and $g \circ f$, and their domains. Domain = all real numbers

$$f(x) = x^2 - 1, \qquad g(x) = 2x + 1$$

$$f \circ g(x) = f(g(x)) = f(2x+1) = (2x+1)^2 - 1$$
$$= 4x^2 + 4x + 1 - 1 = 4x^2 + 4x$$
$$g \circ f(x) = g(f(x)) = g(x^2 - 1) = 2(x^2-1) - 1$$
$$= 2x^2 - 2 - 1 = 2x^2 - 3$$

11. Find $f \circ g$ and $g \circ f$, and their domains. Domain = all real numbers

$$f(x) = x - 2, \qquad g(x) = x^2 + 3x + 4$$

$$f \circ g(x) = f(g(x)) = f(x^2 + 3x + 4) = x^2 + 3x + 4 - 2$$
$$= x^2 + 3x + 2$$
$$g \circ f(x) = g(f(x)) = g(x-2) = (x-2)^2 + 3(x-2) + 4$$
$$= x^2 - 4x + 4 + 3x - 6 + 4 = x^2 - x + 2$$

12. Find $f \circ g$ and $g \circ f$, and their domains. Domain = all real numbers

$$f(x) = 1 - 3x, \qquad g(x) = \cos x$$

$$f \circ g(x) = f(g(x)) = f(\cos x) = 1 - 3\cos x$$
$$g \circ f(x) = g(f(x)) = g(1-3x) = \cos(1-3x)$$

13. Find $f \circ g$ and $g \circ f$, and their domains.

$$f(x) = x^{1/2}, \qquad g(x) = (1-x)^{1/3}$$

Where will restrictions on the domain come from?

If we ever have $(\text{something})^{1/2}$, say $w^{1/2}$,

then we must have $w \geq 0$.

By contrast, $(\text{something})^{1/3}$, say $w^{1/3}$,

is always defined for all w.

$$f \circ g(x) = f(g(x)) = f\left((1-x)^{1/3}\right)$$

$$= \left((1-x)^{1/3}\right)^{1/2} = (1-x)^{1/6}$$

Must have $(1-x)^{1/3} \geq 0$ which is equivalent to $1-x \geq 0$

$1 \geq x$

$x \leq 1$

Domain of $f \circ g$ is $x \leq 1$ or $(-\infty, 1]$

$$g \circ f(x) = g(f(x)) = g(x^{1/2})$$

$$= \left(1 - x^{1/2}\right)^{1/3}$$

Since formula contains $x^{1/2}$, we must have $x \geq 0$.

Domain of $g \circ f$ is $x \geq 0$ or $[0, \infty)$

14. Find the exact value of $\log_5(125)$. Note: $5^3 = 5 \cdot 5 \cdot 5 = 125$

$$\log_5(5^3) = 3$$

15. Find the exact value of $\log_3(1/27)$.

$$\log_3\left(\frac{1}{3^3}\right) = \log_3\left(3^{-3}\right) = -3$$

16. Find the exact value of $\ln(1/e)$.

$$\ln\left(\frac{1}{e}\right) = \ln\left(e^{-1}\right) = -1$$

17. Find the exact value of $\log_{10}(\sqrt{10})$.

$$\log_{10}\left(10^{1/2}\right) = \frac{1}{2}$$

18. Find the exact value of $\log_2(6) - \log_2(15) + \log_2(20)$.

$$\log_2(2 \cdot 3) - \log_2(3 \cdot 5) + \log_2(4 \cdot 5)$$

$$= \log_2 2 + \log_2 3 - (\log_2 3 + \log_2 5) + \log_2 4 + \log_2 5$$

$$= \log_2 2 + \log_2 4$$

$$= 1 + 2 = 3$$

19. Solve the equation for x.

$$e^{7-4x} = 6$$

$$\ln\left(e^{7-4x}\right) = \ln 6$$

$$7 - 4x = \ln 6$$

$$-4x = \ln 6 - 7$$

$$x = \frac{\ln 6 - 7}{-4} \quad \text{or} \quad \frac{7 - \ln 6}{4}$$

20. Solve the equation for x.

$$\ln(3x - 10) = 2$$

$$e^{\ln(3x-10)} = e^2$$

$$3x - 10 = e^2$$

$$3x = e^2 + 10$$

$$x = \frac{e^2 + 10}{3}$$

21. Solve the equation for x.

$$\ln(x^2 - 1) = 3$$

$$e^{\ln(x^2-1)} = e^3$$

$$x^2 - 1 = e^3$$

$$x^2 = e^3 + 1$$

$$x = \pm\sqrt{e^3+1}$$

22. Solve the equation for x.

$$2^{x-5} = 3$$

$$\ln\left(2^{x-5}\right) = \ln 3$$

$$(x-5)\ln 2 = \ln 3$$

$$x - 5 = \frac{\ln 3}{\ln 2}$$

$$x = 5 + \frac{\ln 3}{\ln 2}$$

OR: $\log_2\left(2^{x-5}\right) = \log_2 3$

$$x - 5 = \log_2 3 \Rightarrow x = 5 + \log_2 3$$

23. Find a formula for the nth term of the sequence.

$$1, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \frac{1}{9}, \ldots$$

$$a_n = \frac{1}{2n-1}$$ if we start at $n=1$

$$a_n = \frac{1}{2n+1}$$ if we start at $n=0$

24. Find a formula for the nth term of the sequence.

$$1, -\frac{1}{3}, \frac{1}{9}, -\frac{1}{27}, \frac{1}{81}, \ldots$$

$$a_n = \frac{(-1)^{n+1}}{3^{n-1}} \text{ or } \frac{(-1)^{n-1}}{3^{n-1}} \text{ or } \left(\frac{-1}{3}\right)^{n-1}$$

if we start at $n=1$

Or $$a_n = \frac{(-1)^n}{3^n} \text{ or } \left(-\frac{1}{3}\right)^n$$ if we start at $n=0$

25. Find a formula for the nth term of the sequence.

$$5, 8, 11, 14, 17, \ldots$$

$a_n = 3n + 2$ if we start at $n = 1$

$a_n = 3n + 5$ if we start at $n = 0$

Also allowed: $a_n = 3n - 1$ starting at $n = 2$

26. Find the first five terms of the recursive sequence.

$$a_0 = 2$$
$$a_1 = 1$$
$$a_n = a_{n-1} + 6a_{n-2} \qquad \text{if } n \geq 3$$

$a_0 = 2$

$a_1 = 1$

$a_2 = a_1 + 6a_0 = 1 + 6 \cdot 2 = 1 + 12 = 13$

$a_3 = a_2 + 6a_1 = 13 + 6 \cdot 1 = 13 + 6 = 19$

$a_4 = a_3 + 6a_2 = 19 + 6 \cdot 13 = 19 + 78 = 97$