MAC2241
Suggested problems on Chapter 2 material
(limits)

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1. Evaluate the limit.

$$\lim_{n\to\infty} \frac{1}{3n^4} = 0 \quad \text{Of the form } \frac{1}{\infty}$$

$$\frac{1}{3n^4} = \frac{1}{3} \cdot \frac{1}{n^4}$$

2. Evaluate the limit.

$$\lim_{n\to\infty} \frac{5}{3^n} = 0 \quad \text{Of the form } \frac{5}{\infty}$$

$$\frac{5}{3^n} = 5 \cdot \frac{1}{3^n} = 5 \cdot \left(\frac{1}{3}\right)^n$$

3. Evaluate the limit.

$$\lim_{n\to\infty} \frac{3+5n}{2+7n} = \frac{5}{7}$$

$$\lim_{n\to\infty} \frac{3+5n}{2+7n} \cdot \frac{\frac{1}{n}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{\frac{3}{n}+5}{\frac{2}{n}+7} = \frac{0+5}{0+7} = \frac{5}{7}$$

4. Evaluate the limit.

$$\lim_{n\to\infty} 1 - (0.2)^n = 1 - 0 = 1$$

$\lim_{n\to\infty} (0.2)^n = 0$ because 0.2 is a constant between 0 and 1

5. Evaluate the limit.

$$\lim_{n\to\infty} 2^{-n} + 6^{-n} = 0$$

$$2^{-n} + 6^{-n} = \frac{1}{2^n} + \frac{1}{6^n} = \left(\frac{1}{2}\right)^n + \left(\frac{1}{6}\right)^n$$

$\frac{1}{2}$ and $\frac{1}{6}$ are constants between 0 and 1

6. Evaluate the limit.

$$\lim_{n\to\infty} \frac{n^2}{\sqrt{n^3+4n}}$$ Bottom grows like $\sqrt{n^3} = n^{3/2}$

$$\lim_{n\to\infty} \frac{n^2}{\sqrt{n^3 + \text{smaller}}} = \lim_{n\to\infty} \frac{n^2}{\sqrt{n^3}} = \lim_{n\to\infty} \frac{n^2}{n^{3/2}} = \lim_{n\to\infty} n^{1/2} = \infty$$

7. Evaluate the limit.

$$\lim_{n\to\infty} \frac{\pi^n}{3^n} = \infty$$

$\lim_{n\to\infty} \left(\frac{\pi}{3}\right)^n = \infty$ because $\frac{\pi}{3}$ is a constant greater than 1

8. Evaluate the limit.

$$\lim_{n\to\infty} \frac{3^{n+2}}{5^n} = 0$$

$$\lim_{n\to\infty} \frac{3^n 3^2}{5^n} = 3^2 \lim_{n\to\infty} \frac{3^n}{5^n} = 3^2 \lim_{n\to\infty} \left(\frac{3}{5}\right)^n$$

$$= 3^2 \cdot 0 = 0$$

because $\frac{3}{5}$ is a constant between 0 and 1

9. Evaluate the limit.

$$\lim_{x\to\infty} \frac{1-x^2}{x^3-x+1} = 0$$

$$\lim_{x\to\infty} \frac{1-x^2}{x^3-x+1} \cdot \frac{\frac{1}{x^3}}{\frac{1}{x^3}} = \lim_{x\to\infty} \frac{\frac{1}{x^3} - \frac{1}{x}}{1 - \frac{1}{x^2} + \frac{1}{x^3}}$$

$$= \frac{0-0}{1-0+0} = \frac{0}{1} = 0$$

10. Evaluate the limit.

$$\lim_{x\to-\infty} 0.6^x \quad \text{Note } x \to -\infty$$

Same as $\lim\limits_{t\to\infty} 0.6^{-t} = \lim\limits_{t\to\infty} \frac{1}{0.6^t} = \lim\limits_{t\to\infty} \left(\frac{1}{0.6}\right)^t$

$= \lim\limits_{t\to\infty} \left(\frac{10}{6}\right)^t = +\infty$ because $\frac{10}{6}$ is a constant greater than 1

11. Evaluate the limit.

$$\lim_{x\to\infty} \frac{x - x\sqrt{x}}{2x^{3/2}+3x-5} = -\frac{1}{2}$$

$$\lim_{x\to\infty} \frac{x - x^{3/2}}{2x^{3/2}+3x-5} \cdot \frac{\frac{1}{x^{3/2}}}{\frac{1}{x^{3/2}}}$$

$$= \lim_{x\to\infty} \frac{\frac{1}{x^{1/2}} - 1}{2 + \frac{3}{x^{1/2}} - \frac{5}{x^{3/2}}} = \frac{0-1}{2+0-0} = \frac{-1}{2}$$

12. Evaluate the limit.

$$\lim_{x\to\infty} \frac{(2x^2+1)^2}{(x-1)^2(x^2+x)}$$

$$\lim_{x\to\infty} \frac{(2x^2+\text{smaller})^2}{(x-\text{smaller})^2(x^2+\text{smaller})} = \lim_{x\to\infty} \frac{(2x^2)^2}{(x)^2\cdot x^2}$$

$$= \lim_{x\to\infty} \frac{4x^4}{x^4} = 4$$

13. Evaluate the limit.

$$\lim_{x\to\infty} \frac{x^2}{\sqrt{x^4+1}} \quad = 1$$

$$\lim_{x\to\infty} \frac{x^2}{\sqrt{x^4+\text{smaller}}} = \lim_{x\to\infty} \frac{x^2}{\sqrt{x^4}} = \lim_{x\to\infty} \frac{x^2}{x^2} = 1$$

14. Evaluate the limit.

$$\lim_{x\to\infty} \left(\sqrt{9x^2+x} - 3x\right)$$

$$\lim_{x\to\infty} \frac{\left(\sqrt{9x^2+x} - 3x\right)\left(\sqrt{9x^2+x} + 3x\right)}{\sqrt{9x^2+x} + 3x}$$

$$= \lim_{x\to\infty} \frac{\left(\sqrt{9x^2+x}\right)^2 - (3x)^2}{\sqrt{9x^2+x} + 3x} = \lim_{x\to\infty} \frac{9x^2+x-9x^2}{\sqrt{9x^2+x} + 3x}$$

$$= \lim_{x\to\infty} \frac{x}{\sqrt{9x^2+\text{smaller}} + 3x} = \lim_{x\to\infty} \frac{x}{3x+3x} = \lim_{x\to\infty} \frac{x}{6x} = \frac{1}{6}$$

15. Determine whether the limit is $+\infty$ or $-\infty$.

$$\lim_{x \to -3^+} \frac{x+2}{x+3}$$

$x \to -3^+ \Rightarrow x > -3 \Rightarrow x+3 > 0$

$$\frac{x+2}{x+3} = \frac{\text{near } -1 \text{ and negative}}{\text{near } 0 \text{ and positive}}$$

Limit is $-\infty$.

16. Determine whether the limit is $+\infty$ or $-\infty$.

$$\lim_{x \to -3^-} \frac{x+2}{x+3}$$

$x \to -3^- \Rightarrow x < -3 \Rightarrow x+3 < 0$

$$\frac{x+2}{x+3} = \frac{\text{near } -1 \text{ and negative}}{\text{near } 0 \text{ and positive}}$$

Limit is $+\infty$.

17. Determine whether the limit is $+\infty$ or $-\infty$.

$$\lim_{x \to 1} \frac{2-x}{(x-1)^2}$$

Top is near 1 and positive

Bottom is near 0 and positive

Limit is $+\infty$.

18. Determine whether the limit is $+\infty$ or $-\infty$.

$$\lim_{x \to 5^-} \frac{e^x}{(x-5)^3}$$

Top is near e^5, which is positive

$x < 5 \Rightarrow x - 5 < 0 \Rightarrow (x-5)^3 < 0$

Bottom is near 0 and negative

Limit is $-\infty$

19. Determine whether the limit is $+\infty$ or $-\infty$.

$$\lim_{x \to 3^+} \ln(x^2 - 9)$$

$x \to 3^+ \Rightarrow x > 3$ and x approaches 3

$\Rightarrow x^2$ is slightly greater than 9 $\Rightarrow x^2 - 9$ slightly greater than 0

By our knowledge of the logarithm function,

We know $\ln$(something slightly more than 0) approaches $-\infty$.

20. Determine whether the limit is $+\infty$ or $-\infty$.

$$\lim_{x \to 2^-} \frac{x^2 - 2x}{x^2 - 4x + 4}$$

$$\lim_{x \to 2^-} \frac{x(x-2)}{(x-2)^2} = \lim_{x \to 2^-} \frac{x}{x-2}$$

Top is near 2 and positive

Bottom is near 0 and negative

Limit is $-\infty$.

21. Evaluate the limit.

$$\lim_{x \to 5} \frac{x^2 - 6x + 5}{x - 5}$$

$$\lim_{x \to 5} \frac{(x-1)(x-5)}{x-5} = \lim_{x \to 5} (x-1) = 5-1 = 4$$

22. Evaluate the limit.

$$\lim_{x \to 4} \frac{x^2 - 4x}{x^2 - 3x - 4}$$

$$\lim_{x \to 4} \frac{x(x-4)}{(x+1)(x-4)} = \lim_{x \to 4} \frac{x}{x+1} = \frac{4}{5}$$

23. Evaluate the limit.

$$\lim_{t \to -3} \frac{t^2 - 9}{2t^2 + 7t + 3}$$

$$\lim_{t \to -3} \frac{(t+3)(t-3)}{(t+3)(2t+1)} = \lim_{t \to -3} \frac{t-3}{2t+1}$$

$$= \frac{-3-3}{-6+1} = \frac{-6}{-5} = \frac{6}{5}.$$

24. Evaluate the limit.

$$\lim_{h\to 0} \frac{(4+h)^2-16}{h}$$

$$\lim_{h\to 0} \frac{16+8h+h^2-16}{h} = \lim_{h\to 0} \frac{8h+h^2}{h} = \lim_{h\to 0} \frac{(8+h)h}{h}$$

$$= \lim_{h\to 0} (8+h) = 8+0 = 8$$

25. Evaluate the limit.

$$\lim_{h\to 0} \frac{(2+h)^3-8}{h}$$

Note: $(a+b)^3 = a^3+3a^2b+3ab^2+b^3$

$2^3 \quad 3\cdot 2^2\cdot h \quad 3\cdot 2\cdot h^2$

$$\lim_{h\to 0} \frac{8+12h+6h^2+h^3-8}{h} = \lim_{h\to 0} \frac{12h+6h^2+h^3}{h}$$

$$= \lim_{h\to 0} \frac{(12+6h+h^2)h}{h} = \lim_{h\to 0} (12+6h+h^2) = 12+0+0 = 12$$

26. Evaluate the limit.

$$\lim_{h\to 0} \frac{\sqrt{1+h}-1}{h}$$

$$\lim_{h\to 0} \frac{\sqrt{1+h}-1}{h} \cdot \frac{\sqrt{1+h}+1}{\sqrt{1+h}+1} = \lim_{h\to 0} \frac{(\sqrt{1+h})^2-1^2}{h(\sqrt{1+h}+1)}$$

$$= \lim_{h\to 0} \frac{1+h-1}{h(\sqrt{1+h}+1)} = \lim_{h\to 0} \frac{h}{h(\sqrt{1+h}+1)} = \lim_{h\to 0} \frac{1}{\sqrt{1+h}+1}$$

$$= \frac{1}{\sqrt{1+0}+1} = \frac{1}{\sqrt{1}+1} = \frac{1}{1+1} = \frac{1}{2}$$

27. Evaluate the limit.

$$\lim_{x\to -4} \frac{\frac{1}{4}+\frac{1}{x}}{4+x}$$

$$\lim_{x\to -4} \frac{\frac{1}{4}+\frac{1}{x}}{4+x} \cdot \frac{4x}{4x} = \lim_{x\to -4} \frac{\frac{4x}{4}+\frac{4x}{x}}{(4+x)\cdot 4x}$$

$$= \lim_{x\to -4} \frac{x+4}{(4+x)\cdot 4x} = \lim_{x\to -4} \frac{1}{4x} = \frac{1}{4\cdot(-4)} = -\frac{1}{16}$$

28. Evaluate the limit.

$$\lim_{x\to -1} \frac{x^2+2x+1}{x^4-1}$$

$$\lim_{x\to -1} \frac{(x+1)^2}{(x^2+1)(x^2-1)} = \lim_{x\to -1} \frac{(x+1)^2}{(x^2+1)(x+1)(x-1)}$$

$$= \lim_{x\to -1} \frac{x+1}{(x^2+1)(x-1)} = \frac{-1+1}{((-1)^2+1)(-1-1)} = \frac{0}{2\cdot(-2)}$$

$$= 0$$

29. Evaluate the limit.

$$\lim_{x\to 16} \frac{4-\sqrt{x}}{16x-x^2}$$

$$\lim_{x\to 16} \frac{4-\sqrt{x}}{16x-x^2}\cdot\frac{4+\sqrt{x}}{4+\sqrt{x}} = \lim_{x\to 16} \frac{4^2-(\sqrt{x})^2}{(16x-x^2)(4+\sqrt{x})}$$

$$= \lim_{x\to 16} \frac{16-x}{(16-x)\cdot x\cdot(4+\sqrt{x})} = \lim_{x\to 16} \frac{1}{x\cdot(4+\sqrt{x})}$$

$$= \frac{1}{16\cdot(4+\sqrt{16})} = \frac{1}{16\cdot(4+4)} = \frac{1}{16\cdot 8} = \frac{1}{128}$$

30. Evaluate the limit.

$$\lim_{t\to 0}\left(\frac{1}{t}-\frac{1}{t^2+t}\right)$$

$$\lim_{t\to 0}\left(\frac{1}{t}-\frac{1}{t(t+1)}\right) = \lim_{t\to 0}\left(\frac{t+1}{t(t+1)}-\frac{1}{t(t+1)}\right)$$

$$= \lim_{t\to 0}\frac{t+1-1}{t(t+1)} = \lim_{t\to 0}\frac{t}{t(t+1)} = \lim_{t\to 0}\frac{1}{t+1}$$

$$= \frac{1}{0+1} = \frac{1}{1} = 1.$$