MAC2241
Suggested problems on Chapter 3 material (derivatives)

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1. Find the derivative of the function using the definition of derivative.

$$f(x) = \frac{1}{2}x - \frac{1}{3}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\left(\frac{1}{2}(x+h) - \frac{1}{3}\right) - \left(\frac{1}{2}x - \frac{1}{3}\right)}{h} = \lim_{h \to 0} \frac{\frac{1}{2}x + \frac{1}{2}h - \frac{1}{3} - \frac{1}{2}x + \frac{1}{3}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{1}{2}h}{h} = \lim_{h \to 0} \frac{1}{2} = \frac{1}{2}.$$

2. Find the derivative of the function using the definition of derivative.

$$f(x) = 5x - 9x^2$$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\left(5(x+h) - 9(x+h)^2\right) - (5x - 9x^2)}{h}$$

$$= \lim_{h\to 0} \frac{5x + 5h - 9(x^2 + 2xh + h^2) - 5x + 9x^2}{h}$$

$$= \lim_{h\to 0} \frac{5x + 5h - 9x^2 - 18xh - 9h^2 - 5x + 9x^2}{h}$$

$$= \lim_{h\to 0} \frac{5h - 18xh - 9h^2}{h} = \lim_{h\to 0} \frac{(5 - 18x - 9h)h}{h}$$

$$= \lim_{h\to 0} (5 - 18x - 9h) = 5 - 18x - 0$$

$$= 5 - 18x$$

3. Find the derivative of the function using the definition of derivative.

$$f(x) = \frac{1}{\sqrt{x}}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}}}{h} \cdot \frac{\sqrt{x} \cdot \sqrt{x+h}}{\sqrt{x} \cdot \sqrt{x+h}}$$

$$= \lim_{h \to 0} \frac{\frac{\sqrt{x} \cdot \sqrt{x+h}}{\sqrt{x+h}} - \frac{\sqrt{x} \cdot \sqrt{x+h}}{\sqrt{x}}}{h \cdot \sqrt{x} \cdot \sqrt{x+h}} = \lim_{h \to 0} \frac{\sqrt{x} - \sqrt{x+h}}{h \cdot \sqrt{x} \cdot \sqrt{x+h}}$$

$$= \lim_{h \to 0} \frac{\sqrt{x} - \sqrt{x+h}}{h \cdot \sqrt{x} \cdot \sqrt{x+h}} \cdot \frac{\sqrt{x} + \sqrt{x+h}}{\sqrt{x} + \sqrt{x+h}}$$

$$= \lim_{h \to 0} \frac{(\sqrt{x})^2 - (\sqrt{x+h})^2}{h\sqrt{x}\sqrt{x+h}\,(\sqrt{x} + \sqrt{x+h})} = \lim_{h \to 0} \frac{x - (x+h)}{h\sqrt{x}\sqrt{x+h}\,(\sqrt{x} + \sqrt{x+h})}$$

$$= \lim_{h \to 0} \frac{-h}{h\sqrt{x}\sqrt{x+h}\,(\sqrt{x} + \sqrt{x+h})} = \lim_{h \to 0} \frac{-1}{\sqrt{x}\sqrt{x+h}\,(\sqrt{x} + \sqrt{x+h})}$$

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$$= \frac{-1}{\sqrt{x}\sqrt{x+0}\left(\sqrt{x}+\sqrt{x+0}\right)}$$

$$= \frac{-1}{\sqrt{x}\cdot\sqrt{x}\left(\sqrt{x}+\sqrt{x}\right)} = \frac{-1}{\sqrt{x}\cdot\sqrt{x}\cdot 2\sqrt{x}}$$

$$= \frac{-1}{2x^{3/2}}$$

4. Find the derivative of the function using the definition of derivative.

$$f(x) = x^4$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{(x+h)^4 - x^4}{h}$$

FACT: $(x+h)^4 = x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4$

"Pascal's triangle"

$$\begin{array}{ccccccccc} & & & & 1 & & & & \\ & & & 1 & & 1 & & & \\ & & 1 & & 2 & & 1 & & \\ & 1 & & 3 & & 3 & & 1 & \\ 1 & & 4 & & 6 & & 4 & & 1 \end{array}$$

$(x+h)^2 = x^2 + 2xh + h^2$

$(x+h)^3 = x^3 + 3x^2h + 3xh^2 + h^3$

$$f'(x) = \lim_{h \to 0} \frac{x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4 - x^4}{h}$$

$$= \lim_{h \to 0} \frac{4x^3h + 6x^2h^2 + 4xh^3 + h^4}{h}$$

$$= \lim_{h \to 0} \frac{(4x^3 + 6x^2h + 4xh^2 + h^3)h}{h}$$

$$= \lim_{h \to 0} (4x^3 + 6x^2h + 4xh^2 + h^3)$$

$$= 4x^3 + 0 + 0 + 0 \quad = 4x^3$$