

WRITE YOUR NAME:

MAC 2241 Homework 3

Due in class, Monday September 17th

You can use more paper if necessary, but please STAPLE

Question 1. Find the exact value of each expression.

- $\ln(1/e)$
- $e^{-2\ln 5}$
- $\ln(\ln e^{e^{10}})$

$$(i) \ln(1/e) = \ln(e^{-1}) = -1$$

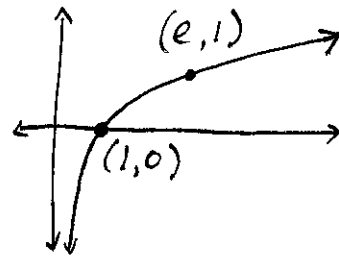
$$(ii) e^{-2\ln 5} = (e^{\ln 5})^{-2} = 5^{-2} = \frac{1}{5^2} = \frac{1}{25}$$

$$(iii) \text{ First, } \ln e^w = w$$

$$\text{So } \ln e^{e^{10}} = e^{10}$$

$$\text{Therefore } \ln(\ln e^{e^{10}}) = \ln(e^{10}) = 10$$

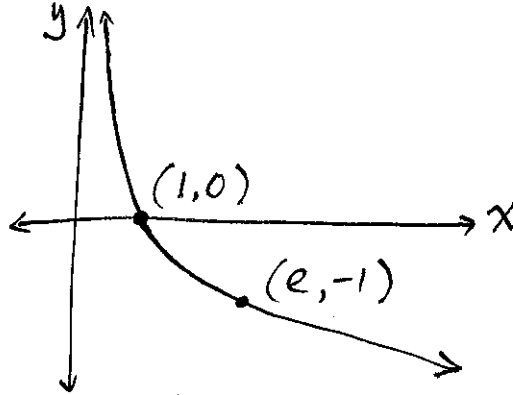
"parent" function $y = \ln x$



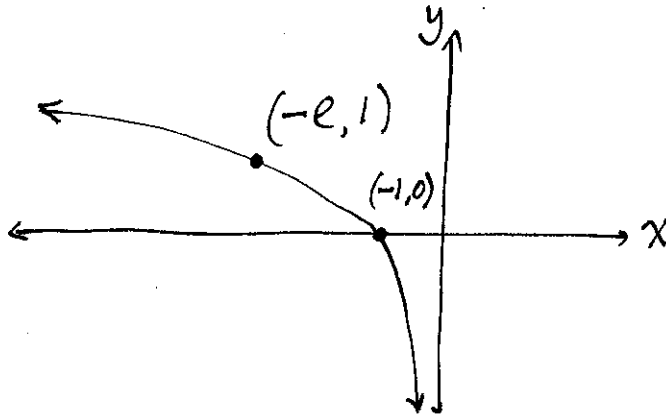
Question 2. Make a rough sketch of the graph of each function.

- $y = -\ln x$
- $y = \ln(-x)$
- $y = \ln|x|$

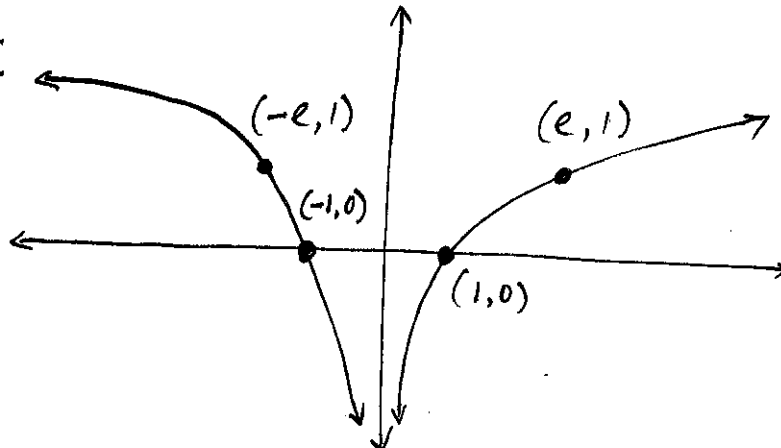
$y = -\ln x$:



$y = \ln(-x)$:



$y = \ln|x|$:



Question 3. Solve each equation for x .

- $e^{7-4x} = 6$

- $\ln(3x - 10) = 2$

- $\ln(x^2 - 1) = 3$

(i) $e^{7-4x} = 6$

$$\ln(e^{7-4x}) = \ln 6$$

$$7 - 4x = \ln 6$$

$$-4x = \ln 6 - 7$$

$$4x = 7 - \ln 6$$

$$x = \frac{7 - \ln 6}{4}$$

(ii) $\ln(3x - 10) = 2$

$$e^{\ln(3x-10)} = e^2$$

$$3x - 10 = e^2$$

$$3x = e^2 + 10$$

$$x = \frac{e^2 + 10}{3}$$

(iii) $\ln(x^2 - 1) = 3$

$$e^{\ln(x^2-1)} = e^3$$

$$x^2 - 1 = e^3$$

$$x^2 = 1 + e^3$$

$$x = \pm \sqrt{1 + e^3}$$

Facts: The functions $f(x) = e^x$ and $f(x) = \ln x$ are both increasing. So we can conclude

$$a < b \Rightarrow e^a < e^b$$

$$a < b \Rightarrow \ln a < \ln b$$

Question 4. Solve each inequality for x .

- $\ln x < 0$
- $e^x > 5$
- $1 < e^{3x-1} < 2$

$$(i) \quad \ln x < 0$$

$$e^{\ln x} < e^0$$

$$x < 1$$

And also $x > 0$ in order for $\ln x$ to be defined

$$(ii) \quad e^x > 5$$

$$\ln(e^x) > \ln 5$$

$$x > \ln 5$$

$$(iii) \quad 1 < e^{3x-1} < 2$$

$$\ln 1 < \ln(e^{3x-1}) < \ln 2$$

$$0 < 3x - 1 < \ln 2$$

$$1 < 3x < 1 + \ln 2$$

$$\frac{1}{3} < x < \frac{1 + \ln 2}{3}$$