WRITE YOUR NAME:

MAC 2241 Section U01 Test 1
Monday February 13th
Total possible score: 20 points (2 points per page)

Question 1. If $f(x) = \frac{1}{x}$, simplify the expression.

$$\frac{f(7+h) - f(7)}{h}$$

$$\frac{\frac{1}{7+h} - \frac{1}{7}}{h} \cdot \frac{7(7+h)}{7(7+h)}$$

$$= \frac{\frac{7(7+h)}{7+h} - \frac{7(7+h)}{7}}{h \cdot 7(7+h)} = \frac{7-(7+h)}{h \cdot 7(7+h)}$$

$$= \frac{7-7-h}{h \cdot 7(7+h)} = \frac{-h}{h \cdot 7(7+h)}$$

$$= \frac{-1}{7(7+h)}$$

Question 2. Find the domain of the function.

$$f(x) = \frac{1}{(x-3)(x-8)} + \sqrt{x-5}$$

Since there is division by $x-3$, must have $x-3 \neq 0$

$x \neq 3$

Since there is division by $x-8$, must have $x-8 \neq 0$

$x \neq 8$

Since there is the square root of $x-5$,

must have $x-5 \geq 0$

$x \geq 5$

Conclusion: Must have $x \geq 5$ and also cannot have $x = 3$ (already forbidden by $x \geq 5$)

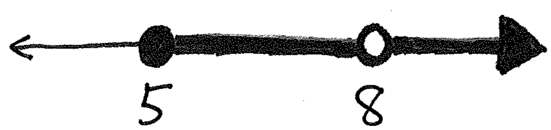
and also cannot have $x = 8$

Domain: $[5, 8) \cup (8, \infty)$

or: $5 \leq x < 8$ or $8 < x$

or: $x \geq 5$, $x \neq 8$

or :



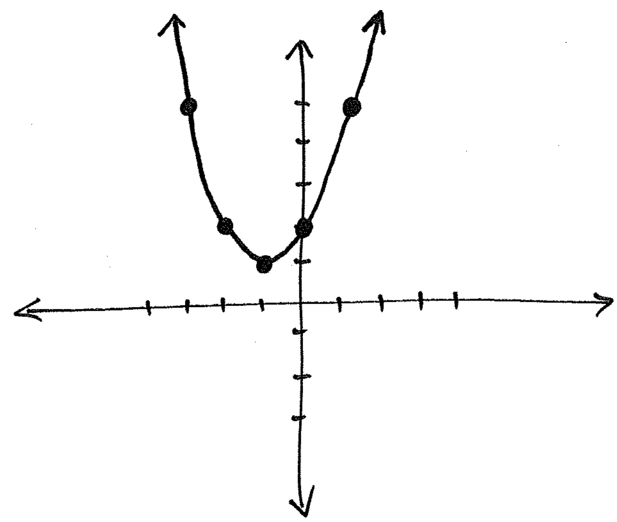
Question 3. Draw a reasonable graph of the function. Label at least **five** points.

$$f(x) = x^2 + 2x + 2$$

It may help to complete the square:

$$f(x) = x^2 + 2x + 1 + 1$$
$$= (x+1)^2 + 1$$

x	$f(x) = (x+1)^2 + 1$	
-4	$(-3)^2 + 1$	$= 10$
-3	$(-2)^2 + 1$	$= 5$
-2	$(-1)^2 + 1$	$= 2$
-1	$(0)^2 + 1$	$= 1$
0	$(1)^2 + 1$	$= 2$
1	$(2)^2 + 1$	$= 5$
2	$(3)^2 + 1$	$= 10$
3	$(4)^2 + 1$	$= 17$
4	$(5)^2 + 1$	$= 26$



Question 4. Find $f \circ g$ and $g \circ f$, and their domains.

$$f(x) = x^2 - 1, \qquad g(x) = 3x + 1$$

$f \circ g$: $f(g(x)) = f(3x+1)$

$= (3x+1)^2 - 1 = 9x^2 + 6x + 1 - 1$

$= 9x^2 + 6x$

$g \circ f$: $g(f(x)) = g(x^2 - 1)$

$= 3(x^2 - 1) + 1 = 3x^2 - 3 + 1$

$= 3x^2 - 2$

Domain in each case: All real numbers

Question 5. Find $f \circ f$ and simplify, and find its domain.

$$f(x) = \frac{1}{x-1}$$

$$f(f(x)) = f\left(\frac{1}{x-1}\right)$$

$$= \frac{1}{\frac{1}{x-1} - 1}$$

$$= \frac{1}{\frac{1}{x-1} - 1} \cdot \frac{x-1}{x-1} \quad \text{if } x \neq 1$$

$$= \frac{x-1}{1-(x-1)} = \frac{x-1}{1-x+1} = \frac{x-1}{2-x}$$

Domain: x cannot be 1 or 2

Question 6. Find the exact value of $\log_5(10) + \log_5(15) - \log_5(6)$.

$$= \log_5\left(\frac{10 \cdot 15}{6}\right)$$

$$= \log_5\left(\frac{2 \cdot 5 \cdot 3 \cdot 5}{2 \cdot 3}\right)$$

$$= \log_5\left(5^2\right)$$

$$= 2$$

Question 7. Solve the equation for x.

$$2^{x-4} = 7$$

$$\ln\left(2^{x-4}\right) = \ln 7$$

$$(x-4)\ln 2 = \ln 7$$

$$x-4 = \frac{\ln 7}{\ln 2}$$

$$x = 4 + \frac{\ln 7}{\ln 2}$$

(We can also use logs other than natural logs)

Question 8. Find a formula for the nth term of the sequence.

$$1, 5, 9, 13, 17, 21, \ldots$$

If we start labeling at 1

$a_1 = 1 \quad a_2 = 5 \quad a_3 = 9 \quad a_4 = 13$ etc.

Like $4n$, but "shifted"

$$a_n = 4n - 3$$

$4-3 = 1$
$8-3 = 5$
$12-3 = 9$
$16-3 = 13$
etc.

Another correct answer:

$a_n = 4n + 1$ if we start at $n=0$

then $a_0 = 0 + 1 = 1$
$a_1 = 4 + 1 = 5$
$a_2 = 8 + 1 = 9$
etc.

Question 9. Evaluate the limit.

$$\lim_{n\to\infty} \frac{e^n}{3^n}$$

$$\lim_{n\to\infty} \left(\frac{e}{3}\right)^n$$

$= 0$ because $\frac{e}{3}$ is a constant between 0 and 1.

Question 10. Evaluate the limit.

$$\lim_{n\to\infty} \frac{2n^2}{\sqrt{9n^4+n}}$$

$$\lim_{n\to\infty} \frac{2n^2}{\sqrt{9n^4+n}} \cdot \frac{\frac{1}{n^2}}{\frac{1}{n^2}}$$

Note: $n^2 = \sqrt{n^4}$

$$= \lim_{n\to\infty} \frac{\frac{2n^2}{n^2}}{\frac{\sqrt{9n^4+n}}{\sqrt{n^4}}} = \lim_{n\to\infty} \frac{\frac{2n^2}{n^2}}{\sqrt{\frac{9n^4+n}{n^4}}}$$

$$= \lim_{n\to\infty} \frac{\frac{2n^2}{n^2}}{\sqrt{\frac{9n^4}{n^4}+\frac{n}{n^4}}} = \lim_{n\to\infty} \frac{2}{\sqrt{9+\frac{1}{n^3}}}$$

$$= \frac{2}{\sqrt{9+0}} = \frac{2}{\sqrt{9}} = \frac{2}{3}$$