MAC2241 Spring 2017
Suggested problems for Test 1
(Test 1 is Monday February 13th, in class)

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1. If $f(x) = x^2$, simplify each of the expressions.

$$\frac{f(3+h)-f(3)}{h}$$

$$\frac{f(x+h)-f(x)}{h}$$

$$\frac{f(3+h)-f(3)}{h} = \frac{(3+h)^2-3^2}{h} = \frac{9+6h+h^2-9}{h}$$

$$= \frac{6h+h^2}{h} = \frac{(6+h)h}{h} = 6+h$$

$$\frac{f(x+h)-f(x)}{h} = \frac{(x+h)^2-x^2}{h} = \frac{x^2+2xh+h^2-x^2}{h}$$

$$= \frac{2xh+h^2}{h} = \frac{(2x+h)h}{h} = 2x+h$$

2. If $f(x) = \frac{1}{x}$, simplify each of the expressions.

$$\frac{f(3+h)-f(3)}{h}$$

$$\frac{f(x+h)-f(x)}{h}$$

$$\frac{f(3+h)-f(3)}{h} = \frac{\frac{1}{3+h} - \frac{1}{3}}{h} \cdot \frac{3(3+h)}{3(3+h)}$$

$$= \frac{\frac{3(3+h)}{3+h} - \frac{3(3+h)}{3}}{h \cdot 3(3+h)} = \frac{3-(3+h)}{h \cdot 3(3+h)} = \frac{3-3-h}{h \cdot 3(3+h)}$$

$$= \frac{-h}{h \cdot 3(3+h)} = \frac{-1}{3(3+h)}$$

$$\frac{f(x+h)-f(x)}{h} = \frac{\frac{1}{x+h} - \frac{1}{x}}{h} \cdot \frac{x(x+h)}{x(x+h)}$$

$$= \frac{\frac{x(x+h)}{x+h} - \frac{x(x+h)}{x}}{h \cdot x(x+h)} = \frac{x-(x+h)}{h \cdot x(x+h)} = \frac{x-x-h}{h \cdot x(x+h)}$$

$$= \frac{-h}{h \cdot x(x+h)} = \frac{-1}{x(x+h)}$$

3. Find the domain of the function.

$$f(x) = \frac{x+4}{x^2-9}$$

Exceptions:

$x^2 - 9 = 0$

$x^2 = 9$

$x = \pm 3$

or $(x+3)(x-3) = 0$

All x except $+3, -3$

or $(-\infty, -3) \cup (-3, 3) \cup (3, \infty)$

4. Find the domain of the function.

$$f(x) = \frac{2x^3-5}{x^2+x-6}$$

$x^2 + x - 6 = (x+3)(x-2)$

All x except $-3, 2$

or $(-\infty, -3) \cup (-3, 2) \cup (2, \infty)$

5. Find the domain of the function.

$$\sqrt{2-\sqrt{x}}$$

Must have $2 - \sqrt{x} \geq 0$

Must also have $x \geq 0$

$2 - \sqrt{x} \geq 0$

$2 \geq \sqrt{x}$

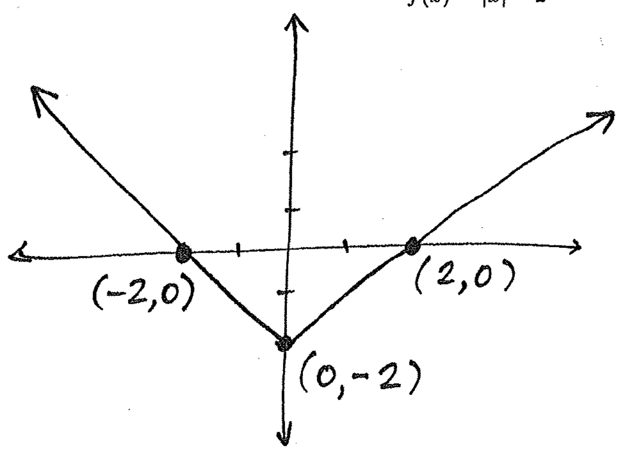
$4 \geq x$

Must have $0 \leq x \leq 4$

$[0, 4]$

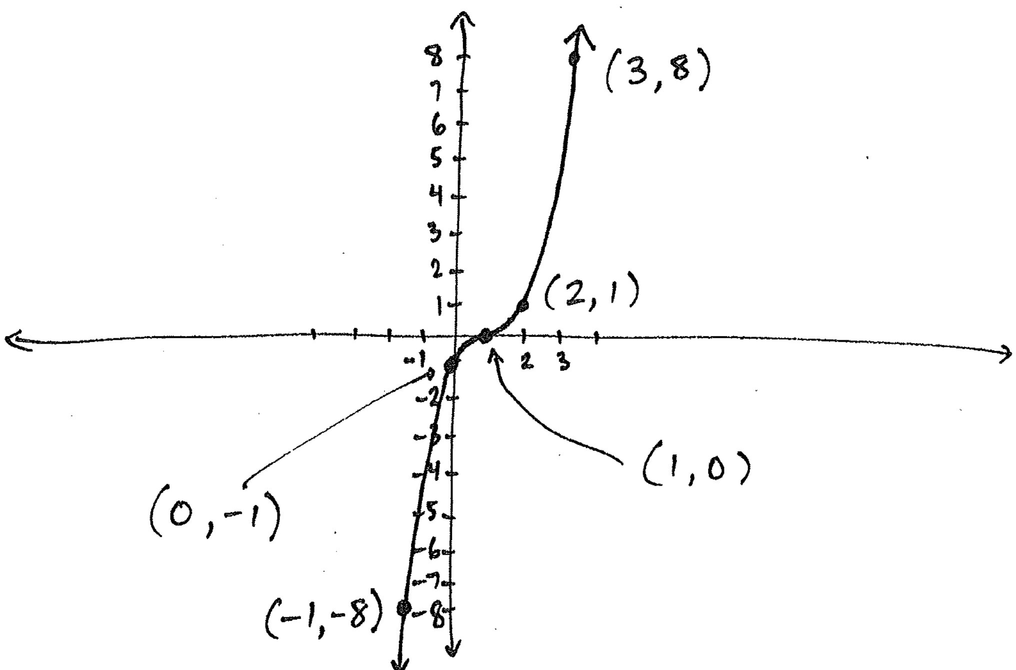
6. Draw a reasonable graph of the function. Label at least three points.

$$f(x) = |x| - 2$$



7. Draw a reasonable graph of the function. Label at least three points.

$$f(x) = (x-1)^3$$

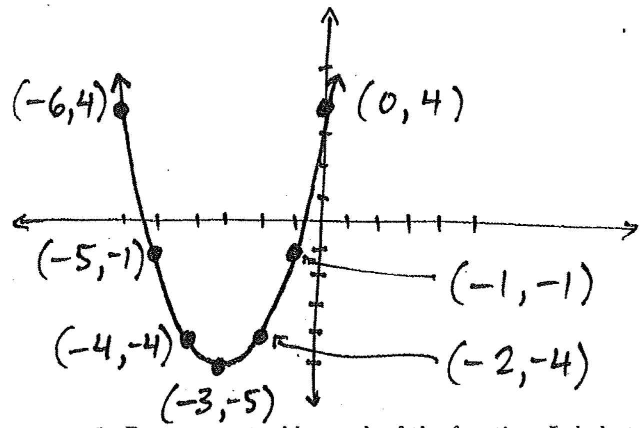


8. Draw a reasonable graph of the function. Label at least three points.

$$f(x) = x^2 + 6x + 4$$

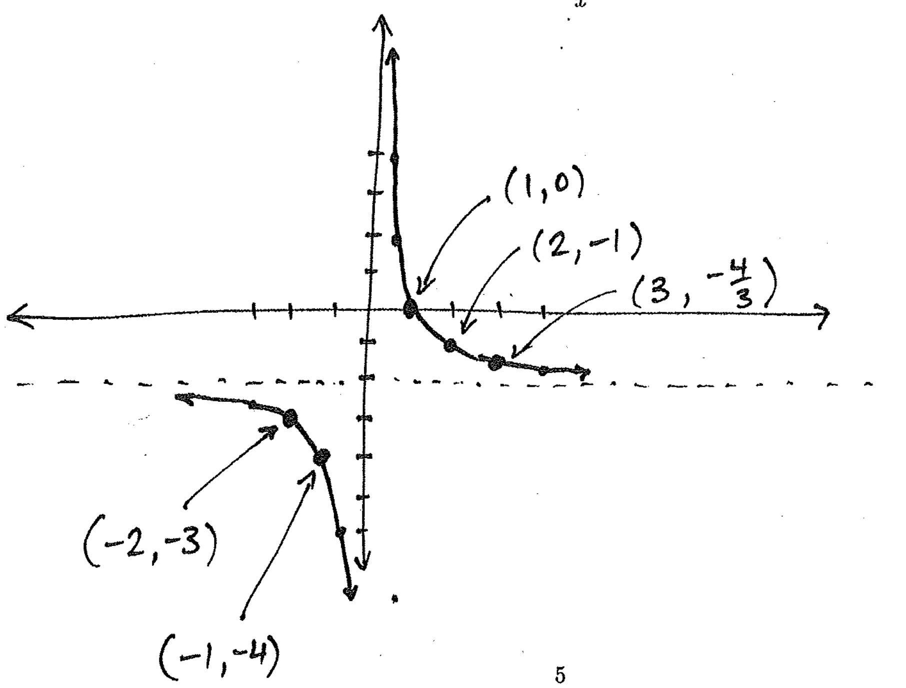
Helpful trick: Complete the square

$f(x) = x^2 + 6x + 9 - 5 = (x+3)^2 - 5$



9. Draw a reasonable graph of the function. Label at least three points.

$$f(x) = \frac{2}{x} - 2$$



10. Find $f \circ g$ and $g \circ f$, and their domains.

$$f(x) = x^2 - 1, \qquad g(x) = 2x + 1$$

$$(f \circ g)(x) = f(g(x)) = f(2x+1)$$
$$= (2x+1)^2 - 1 = 4x^2 + 4x + 1 - 1$$
$$= 4x^2 + 4x$$

$$(g \circ f)(x) = g(f(x)) = g(x^2 - 1)$$
$$= 2(x^2 - 1) + 1 = 2x^2 - 2 + 1 = 2x^2 - 1$$

In both cases, domain is all real numbers

11. Find $f \circ g$ and $g \circ f$, and their domains.

$$f(x) = x - 2, \qquad g(x) = x^2 + 3x + 4$$

$f \circ g$: $f(g(x)) = f(x^2 + 3x + 4)$
$$= (x^2 + 3x + 4) - 2 = x^2 + 3x + 2$$

$g \circ f$: $g(f(x)) = g(x - 2)$
$$= (x-2)^2 + 3(x-2) + 4$$
$$= x^2 - 4x + 4 + 3x - 6 + 4$$
$$= x^2 - x + 2$$

In both cases, domain is all real numbers

12. Find $f \circ g$ and $g \circ f$, and their domains.

$$f(x) = 1 - 3x, \qquad g(x) = \cos x$$

$f \circ g$: $f(g(x)) = f(\cos x) = 1 - 3\cos x$

$g \circ f$: $g(f(x)) = g(1-3x) = \cos(1-3x)$

In both cases domain is all real numbers

13. Find $f \circ g$ and $g \circ f$, and their domains.

$$f(x) = x^{1/2}, \qquad g(x) = (1-x)^{1/3}$$

Domain of f is $x \geq 0$

Domain of g is all real numbers

$f \circ g$: $f(g(x)) = f((1-x)^{1/3})$

$= ((1-x)^{1/3})^{1/2} = (1-x)^{1/6}$

Must have $1-x \geq 0$

$1 \geq x$ Domain is $x \leq 1$

$g \circ f$: $g(f(x)) = g(x^{1/2})$

$= (1-x^{1/2})^{1/3}$ Must have $x \geq 0$

Domain is $x \geq 0$

14. Find the exact value of $\log_5(125)$.

$$125 = 5 \times 5 \times 5 = 5^3$$

$$\log_5(5^3) = 3$$

15. Find the exact value of $\log_3(1/27)$.

$$\log_3(1/27) = \log_3(3^{-3}) = -3$$

16. Find the exact value of $\ln(1/e)$.

$$\ln(1/e) = \ln(e^{-1}) = -1$$

17. Find the exact value of $\log_{10}(\sqrt{10})$.

$$\log_{10}(\sqrt{10}) = \log_{10}(10^{1/2}) = \frac{1}{2}$$

18. Find the exact value of $\log_2(6) - \log_2(15) + \log_2(20)$.

$$= \log_2\left(\frac{6}{15}\right) + \log_2(20)$$

$$= \log_2\left(\frac{6}{15} \cdot 20\right) = \log_2\left(\frac{120}{15}\right)$$

$$= \log_2(8)$$

$$= \log_2(2^3) = 3$$

19. Solve the equation for x.

$$e^{7-4x} = 6$$

$$\ln\left(e^{7-4x}\right) = \ln 6$$

$$7-4x = \ln 6$$

$$-4x = \ln 6 - 7$$

$$x = \frac{\ln 6 - 7}{-4}$$

20. Solve the equation for x.

$$\ln(3x-10) = 2$$

$$e^{\ln(3x-10)} = e^2$$

$$3x - 10 = e^2$$

$$3x = e^2 + 10$$

$$x = \frac{e^2+10}{3}$$

21. Solve the equation for x.

$$\ln(x^2-1) = 3$$

$$e^{\ln(x^2-1)} = e^3$$

$$x^2 - 1 = e^3$$

$$x^2 = e^3 + 1$$

$$x = \pm\sqrt{e^3+1}$$

22. Solve the equation for x.

$$2^{x-5} = 3$$

Method 1. $\log_2\left(2^{x-5}\right) = \log_2 3$

$$x-5 = \log_2 3$$

$$x = 5 + \log_2 3$$

Method 2. $\ln\left(2^{x-5}\right) = \ln 3$

$$(x-5)\ln 2 = \ln 3$$

$$x-5 = \frac{\ln 3}{\ln 2}$$

$$x = 5 + \frac{\ln 3}{\ln 2}$$

23. Find a formula for the nth term of the sequence.

$$1, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \frac{1}{9}, \ldots$$

$$a_n = \frac{1}{2n-1} \quad \text{where } n = 1, 2, 3, \ldots$$

OR $$a_n = \frac{1}{2n+1} \quad \text{where } n = 0, 1, 2, \ldots$$

24. Find a formula for the nth term of the sequence.

$$1, -\frac{1}{3}, \frac{1}{9}, -\frac{1}{27}, \frac{1}{81}, \ldots$$

$$a_n = (-1)^n \cdot \frac{1}{3^n} \qquad n = 0, 1, 2, \ldots$$

25. Find a formula for the nth term of the sequence.

$$5, 8, 11, 14, 17, \ldots$$

$$a_n = 3n + 2 \qquad n = 1, 2, 3, \ldots$$

or $$a_n = 3n + 5 \qquad n = 0, 1, 2, \ldots$$

or $$a_n = 3n - 1 \qquad n = 2, 3, 4, \ldots$$

26. Find the first five terms of the recursive sequence.

$$a_0 = 2$$
$$a_1 = 1$$
$$a_n = a_{n-1} + 6a_{n-2} \qquad \text{if } n \geq 3$$

$a_0 = 2$

$a_1 = 1$

$a_2 = a_1 + 6a_0 = 1 + 6 \cdot 2 = 1 + 12 = 13$

$a_3 = a_2 + 6a_1 = 13 + 6 \cdot 1 = 13 + 6 = 19$

$a_4 = a_3 + 6a_2 = 19 + 6 \cdot 13 = 19 + 78 = 97$

$2, 1, 13, 19, 97$

27. Evaluate the limit.

$$\lim_{n\to\infty} \frac{1}{3n^4} = 0$$

because top is 1 and bottom approaches ∞

28. Evaluate the limit.

$$\lim_{n\to\infty} \frac{5}{3^n}$$

because top is the constant 5 and bottom approaches ∞

29. Evaluate the limit.

$$\lim_{n\to\infty} \frac{3+5n}{2+7n}$$

$$\lim_{n\to\infty} \frac{3+5n}{2+7n} \cdot \frac{\frac{1}{n}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{\frac{3}{n}+5}{\frac{2}{n}+7} = \frac{0+5}{0+7} = \frac{5}{7}$$

30. Evaluate the limit.

$$\lim_{n\to\infty} 1-(0.2)^n$$

Fact: Since 0.2 is a constant between 0 and 1,

we know $\lim_{n\to\infty} 0.2^n = 0$

So the answer to this question is $1-0 = 1$

31. Evaluate the limit.

$$\lim_{n\to\infty} 2^{-n} + 6^{-n}$$

$$= \lim_{n\to\infty} \left(\frac{1}{2^n} + \frac{1}{6^n} \right) = 0 + 0 = 0$$

32. Evaluate the limit.

$$\lim_{n\to\infty} \frac{n^2}{\sqrt{n^3+4n}}$$

Guideline: Top is n^2
Bottom grows like $\sqrt{n^3} = (n^3)^{1/2}$
$= n^{3/2}$

$$\lim_{n\to\infty} \frac{n^2}{\sqrt{n^3+4n}} \cdot \frac{\frac{1}{n^{3/2}}}{\frac{1}{\sqrt{n^3}}} = \lim_{n\to\infty} \frac{n^{1/2}}{\sqrt{1+\frac{4}{n^2}}} = \infty$$

33. Evaluate the limit.

$$\lim_{n\to\infty} \frac{\pi^n}{3^n}$$

$$\lim_{n\to\infty} \left(\frac{\pi}{3}\right)^n = \infty$$ because $\frac{\pi}{3}$ is a constant > 1

34. Evaluate the limit.

$$\lim_{n\to\infty} \frac{3^{n+2}}{5^n}$$

$$\lim_{n\to\infty} \frac{3^n 3^2}{5^n} = \lim_{n\to\infty} 9 \cdot \left(\frac{3}{5}\right)^n$$

$$= 9 \cdot 0 = 0$$

because $\frac{3}{5}$ is a constant between 0 and 1