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MAC 2241 Section U01 Test 2
Monday March 6th
Total possible score: 20 points (2 points per page)

Question 1. Evaluate the limit.

$$\lim_{n\to\infty} \frac{2^n}{e^n}$$

$$\lim_{n\to\infty} \left(\frac{2}{e}\right)^n$$

$= 0$ because $\frac{2}{e}$ is a constant between 0 and 1

$(e \approx 2.7)$

Question 2. Evaluate the limit.

$$\lim_{x\to\infty} \frac{(3x^2+1)^2}{(x^2+1)(x-7)^2}$$

$$\lim_{x\to\infty} \frac{9x^4 + \text{smaller powers}}{(x^2 + \text{smaller})(x^2 + \text{smaller})}$$

$$= \lim_{x\to\infty} \frac{9x^4 + \text{smaller powers}}{x^4 + \text{smaller powers}}$$

$$= \lim_{x\to\infty} \frac{9x^4}{x^4} = \lim_{x\to\infty} 9$$

$$= 9$$

Question 3. Evaluate the limit.

$$\lim_{x\to\infty} \frac{7 - x\sqrt{x}}{17x - x^{3/2}}$$

$$\lim_{x\to\infty} \frac{7 - x^{3/2}}{17x - x^{3/2}} \cdot \frac{\frac{1}{x^{3/2}}}{\frac{1}{x^{3/2}}}$$

$$= \lim_{x\to\infty} \frac{\frac{7}{x^{3/2}} - 1}{\frac{17}{x^{1/2}} - 1}$$

$$= \frac{0 - 1}{0 - 1} = \frac{-1}{-1} = 1$$

Question 4. Evaluate the limit.

$$\lim_{x\to\infty} \left(\sqrt{x^2+10x} - x\right)$$

$$\lim_{x\to\infty} \frac{\left(\sqrt{x^2+10x} - x\right)\left(\sqrt{x^2+10x} + x\right)}{\sqrt{x^2+10x} + x}$$

$$= \lim_{x\to\infty} \frac{\left(\sqrt{x^2+10x}\right)^2 - x^2}{\sqrt{x^2+10x} + x} = \lim_{x\to\infty} \frac{x^2+10x - x^2}{\sqrt{x^2+10x} + x}$$

$$= \lim_{x\to\infty} \frac{10x}{\sqrt{x^2+10x} + x} \cdot \frac{\frac{1}{x}}{\frac{1}{x}}$$

$$= \lim_{x\to\infty} \frac{\frac{10x}{x}}{\frac{\sqrt{x^2+10x}}{\sqrt{x^2}} + \frac{x}{x}} = \lim_{x\to\infty} \frac{10}{\sqrt{\frac{x^2+10x}{x^2}} + 1}$$

$$= \lim_{x\to\infty} \frac{10}{\sqrt{1+\frac{10}{x}} + 1} = \lim_{x\to\infty} \frac{10}{\sqrt{1+0} + 1}$$

$$= \lim_{x\to\infty} \frac{10}{\sqrt{1} + 1} = \lim_{x\to\infty} \frac{10}{1+1} = \frac{10}{2} = 5$$

Question 5. Determine whether the limit is $+\infty$ or $-\infty$.

$$\lim_{x \to 3} \frac{x-4}{(x-3)^2}$$

If x is near 3, then $x-3$ is near 0

$(x-3)^2$ is near 0 and <u>positive</u>

$x-4$ is near -1 and <u>negative</u>

$\dfrac{x-4}{(x-3)^2} \to \dfrac{\text{nonzero}}{\text{zero}}$, so extreme behavior ($+\infty$ or $-\infty$)

$\dfrac{x-4}{(x-3)^2} \to \dfrac{\text{negative}}{\text{positive}}$, so $-\infty$

Answer: $-\infty$

Question 6. Determine whether the limit is $+\infty$ or $-\infty$.

$$\lim_{x \to 4^-} \frac{e^{5x}}{(x-4)^3}$$

$x \to 4^- \Rightarrow x < 4 \Rightarrow x-4$ is negative

$\Rightarrow (x-4)^3$ is negative

When $x \to 4$, $e^{5x} \to e^{5 \cdot 4} = e^{20}$ which is positive (and nonzero)

$$\frac{e^{5x}}{(x-4)^3} \to \frac{\text{nonzero}}{\text{zero}} \quad \text{and} \quad \frac{\text{positive}}{\text{negative}}$$

Answer is $-\infty$

Question 7. Evaluate the limit.

$$\lim_{x \to 5} \frac{x^2 - 7x + 10}{x^2 - 8x + 15}$$

$$\lim_{x \to 5} \frac{(x-2)(x-5)}{(x-3)(x-5)}$$

$$= \lim_{x \to 5} \frac{x-2}{x-3} = \frac{5-2}{5-3}$$

$$= \frac{3}{2}$$

Question 8. Evaluate the limit.

$$\lim_{x \to 3} \frac{\frac{1}{x} - \frac{1}{3}}{x - 3}$$

$$\lim_{x \to 3} \frac{\frac{1}{x} - \frac{1}{3}}{x - 3} \cdot \frac{3x}{3x}$$

$$= \lim_{x \to 3} \frac{\frac{3x}{x} - \frac{3x}{3}}{3x(x-3)} = \lim_{x \to 3} \frac{3 - x}{3x(x-3)}$$

$$= \lim_{x \to 3} \frac{(-1)(x-3)}{3x(x-3)} = \lim_{x \to 3} \frac{-1}{3x}$$

$$= \frac{-1}{9}$$

Question 9. Evaluate the limit.

$$\lim_{x \to 0} \left(\frac{1}{x} - \frac{2}{x^2 + 2x} \right)$$

$$\lim_{x \to 0} \left(\frac{1}{x} - \frac{2}{x(x+2)} \right)$$

$$= \lim_{x \to 0} \left(\frac{x+2}{x(x+2)} - \frac{2}{x(x+2)} \right)$$

$$= \lim_{x \to 0} \frac{x+2-2}{x(x+2)} = \lim_{x \to 0} \frac{x}{x(x+2)}$$

$$= \lim_{x \to 0} \frac{1}{(x+2)} = \frac{1}{0+2} = \frac{1}{2}$$

Question 10.
Find the derivative of the function using the definition of derivative.

$$f(x) = 3x + 17$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{3(x+h) + 17 - (3x + 17)}{h}$$

$$= \lim_{h \to 0} \frac{3x + 3h + 17 - 3x - 17}{h}$$

$$= \lim_{h \to 0} \frac{3h}{h} = \lim_{h \to 0} 3 = 3$$