MAC2241 Spring 2017
Suggested problems for Test 2
(Test 2 is Monday March 6th, in class)

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1. Evaluate the limit.

$$\lim_{n\to\infty} \frac{1}{3n^4} = 0$$

because top is 1 and bottom approaches ∞

2. Evaluate the limit.

$$\lim_{n\to\infty} \frac{5}{3^n} = 0$$

because top is the constant 5 and bottom approaches ∞

3. Evaluate the limit.

$$\lim_{n\to\infty} \frac{3+5n}{2+7n}$$

$$\lim_{n\to\infty} \frac{3+5n}{2+7n} \cdot \frac{\frac{1}{n}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{\frac{3}{n}+5}{\frac{2}{n}+7} = \frac{0+5}{0+7} = \frac{5}{7}$$

4. Evaluate the limit.

$$\lim_{n\to\infty} 1 - (0.2)^n$$

Fact: Since 0.2 is a constant between 0 and 1,

we know $\lim_{n\to\infty} 0.2^n = 0$

Answer to this question is $1 - 0 = 1$

5. Evaluate the limit.

$$\lim_{n\to\infty} 2^{-n} + 6^{-n}$$

$$= \lim_{n\to\infty} \left(\frac{1}{2^n} + \frac{1}{6^n} \right) = 0 + 0 = 0$$

6. Evaluate the limit.

$$\lim_{n\to\infty} \frac{n^2}{\sqrt{n^3 + 4n}}$$

Shortcut: Top is n^2

Bottom grows like $\sqrt{n^3} = (n^3)^{1/2}$

$= n^{3/2}$

Answer is $+\infty$

$$\lim_{n\to\infty} \frac{n^2}{\sqrt{n^3 + 4n}} \cdot \frac{\frac{1}{n^{3/2}}}{\frac{1}{\sqrt{n^3}}} = \lim_{n\to\infty} \frac{n^{1/2}}{\sqrt{1 + \frac{4}{n^2}}} = \infty$$

7. Evaluate the limit.

$$\lim_{n\to\infty} \frac{\pi^n}{3^n}$$

$$\lim_{n\to\infty} \left(\frac{\pi}{3}\right)^n = +\infty$$ because $\frac{\pi}{3}$ is a constant > 1

8. Evaluate the limit.

$$\lim_{n\to\infty} \frac{3^{n+2}}{5^n}$$

$$\lim_{n\to\infty} \frac{3^n 3^2}{5^n} = \lim_{n\to\infty} 9 \cdot \left(\frac{3}{5}\right)^n$$

$$= 9 \cdot 0 = 0$$

because $\frac{3}{5}$ is a constant between 0 and 1

9. Evaluate the limit.

$$\lim_{x\to\infty} \frac{1-x^2}{x^3-x+1}$$

$$\lim_{x\to\infty} \frac{1-x^2}{x^3-x+1} \cdot \frac{\frac{1}{x^3}}{\frac{1}{x^3}} = \lim_{x\to\infty} \frac{\frac{1}{x^3} - \frac{1}{x}}{1 - \frac{1}{x^2} + \frac{1}{x^3}}$$

$$= \frac{0-0}{1-0+0} = \frac{0}{1} = 0$$

10. Evaluate the limit.

$$\lim_{x\to-\infty} 0.6^x$$

↑
NOTE

$$\lim_{x\to-\infty} 0.6^x = \lim_{x\to\infty} 0.6^{-x} = \lim_{x\to\infty} \left(\frac{1}{0.6}\right)^x$$

$$0.6 = \frac{6}{10} = \frac{3}{5} \Rightarrow \frac{1}{0.6} = \frac{5}{3}$$

$$\lim_{x\to\infty} \left(\frac{5}{3}\right)^x = \infty$$

since $\frac{5}{3} > 1$

11. Evaluate the limit.

$$\lim_{x\to\infty} \frac{x - x\sqrt{x}}{2x^{3/2}+3x-5}$$

Note $x\sqrt{x} = x \cdot x^{1/2} = x^{3/2}$

$$\lim_{x\to\infty} \frac{x - x^{3/2}}{2x^{3/2}+3x-5} \cdot \frac{\frac{1}{x^{3/2}}}{\frac{1}{x^{3/2}}}$$

$$= \lim_{x\to\infty} \frac{\frac{1}{x^{1/2}} - 1}{2 + \frac{3}{x^{1/2}} - \frac{5}{x^{3/2}}} = \frac{0-1}{2+0-0} = \frac{-1}{2}$$

12. Evaluate the limit.

$$\lim_{x\to\infty} \frac{(2x^2+1)^2}{(x-1)^2(x^2+x)}$$

$(2x^2+1)^2 = 4x^4$ + smaller powers

$(x-1)^2 = x^2$ + smaller powers

$(x-1)^2(x^2+x) = x^4$ + smaller powers

Answer: $\frac{4}{1} = 4$

13. Evaluate the limit.

$$\lim_{x\to\infty} \frac{x^2}{\sqrt{x^4+1}}$$

Note: Bottom grows like $\sqrt{x^4} = x^2$

$$\lim_{x\to\infty} \frac{x^2}{\sqrt{x^4+1}} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}} = \lim_{x\to\infty} \frac{1}{\sqrt{1+\frac{1}{x^4}}}$$

Using $x^2 = \sqrt{x^4}$

$$= \frac{1}{\sqrt{1+0}} = 1$$

14. Evaluate the limit.

$$\lim_{x\to\infty} \left(\sqrt{9x^2+x} - 3x\right)$$

$$\lim_{x\to\infty} \frac{\left(\sqrt{9x^2+x} - 3x\right)\left(\sqrt{9x^2+x} + 3x\right)}{\sqrt{9x^2+x} + 3x}$$

$$= \lim_{x\to\infty} \frac{\left(\sqrt{9x^2+x}\right)^2 - (3x)^2}{\sqrt{9x^2+x} + 3x} = \lim_{x\to\infty} \frac{9x^2+x-9x^2}{\sqrt{9x^2+x}+3x}$$

$$= \lim_{x\to\infty} \frac{x}{\sqrt{9x^2+x}+3x} \cdot \frac{\frac{1}{x}}{\frac{1}{x}} = \lim_{x\to\infty} \frac{1}{\sqrt{9+\frac{1}{x}}+3}$$

Use $x = \sqrt{x^2}$

$$= \frac{1}{\sqrt{9}+3} = \frac{1}{3+3} = \frac{1}{6}$$

15. Determine whether the limit is $+\infty$ or $-\infty$.

$$\lim_{x \to -3^+} \frac{x+2}{x+3}$$

x near -3 and x greater than -3. $x > -3$

$x+3 > 0$

$x+2$ is near -1, so negative

$x+3$ is near 0 and positive

Answer: $-\infty$

16. Determine whether the limit is $+\infty$ or $-\infty$.

$$\lim_{x \to -3^-} \frac{x+2}{x+3}$$

x near -3 and x less than -3. $x < -3$

$x+3 < 0$

$x+2$ is near -1, so negative

$x+3$ is near 0 and negative

Answer: $+\infty$

17. Determine whether the limit is $+\infty$ or $-\infty$.

$$\lim_{x \to 1} \frac{2-x}{(x-1)^2}$$

x near 1 $\Rightarrow$ $x-1$ is near 0

$(x-1)^2$ is near 0 and positive

$2-x$ is near 1, so positive

Answer: $+\infty$

18. Determine whether the limit is $+\infty$ or $-\infty$.

$$\lim_{x \to 5^-} \frac{e^x}{(x-5)^3}$$

x near 5 and x less than 5. $x < 5$

$x - 5 < 0$

$(x-5)^3$ is near 0 and negative

e^x is near e^5, so positive Answer: $-\infty$

19. Determine whether the limit is $+\infty$ or $-\infty$.

$$\lim_{x \to 3^+} \ln(x^2 - 9)$$

x near 3 and x greater than 3. $x > 3$

$x^2 > 9$

$x^2 - 9 > 0$

From our knowledge of behavior of the logarithm function,

answer is $-\infty$

20. Determine whether the limit is $+\infty$ or $-\infty$.

$$\lim_{x \to 2^-} \frac{x^2 - 2x}{x^2 - 4x + 4}$$

$$\lim_{x \to 2^-} \frac{x(x-2)}{(x-2)(x-2)} = \lim_{x \to 2^-} \frac{x}{x-2}$$

$x < 2$ Top x is near 2, so positive

$x - 2 < 0$ Bottom $x - 2$ is near 0 and negative

Answer is $-\infty$

21. Evaluate the limit.

$$\lim_{x \to 5} \frac{x^2 - 6x + 5}{x - 5}$$

$$\lim_{x \to 5} \frac{(x-5)(x-1)}{x-5} = \lim_{x \to 5} (x-1)$$

$$= 5 - 1 = 4$$

22. Evaluate the limit.

$$\lim_{x \to 4} \frac{x^2 - 4x}{x^2 - 3x - 4}$$

$$\lim_{x \to 4} \frac{x(x-4)}{(x+1)(x-4)} = \lim_{x \to 4} \frac{x}{x+1} = \frac{4}{4+1}$$

$$= \frac{4}{5}$$

23. Evaluate the limit.

$$\lim_{t \to -3} \frac{t^2 - 9}{2t^2 + 7t + 3}$$

$$\lim_{t \to -3} \frac{(t+3)(t-3)}{(t+3)(2t+1)} = \lim_{t \to -3} \frac{t-3}{2t+1}$$

$$= \frac{-3-3}{-6+1} = \frac{-6}{-5} = \frac{6}{5}$$

24. Evaluate the limit.

$$\lim_{h\to 0} \frac{(4+h)^2-16}{h}$$

$$\lim_{h\to 0} \frac{16+8h+h^2-16}{h} = \lim_{h\to 0} \frac{8h+h^2}{h}$$

$$= \lim_{h\to 0} \frac{(8+h)h}{h} = \lim_{h\to 0}(8+h) = 8+0 = 8$$

25. Evaluate the limit.

$$\lim_{h\to 0} \frac{(2+h)^3-8}{h} \qquad (a+b)^3 = a^3+3a^2b+3ab^2+b^3$$

$$\lim_{h\to 0} \frac{8+3\cdot 4h+3\cdot 2h^2+h^3-8}{h} = \lim_{h\to 0} \frac{12h+6h^2+h^3}{h}$$

$$= \lim_{h\to 0} \frac{(12+6h+h^2)h}{h} = \lim_{h\to 0}(12+6h+h^2) = 12+0+0$$

$$= 12$$

26. Evaluate the limit.

$$\lim_{h\to 0} \frac{\sqrt{1+h}-1}{h}$$

$$\lim_{h\to 0} \frac{(\sqrt{1+h}-1)(\sqrt{1+h}+1)}{h(\sqrt{1+h}+1)} = \lim_{h\to 0} \frac{(\sqrt{1+h})^2-1^2}{h(\sqrt{1+h}+1)}$$

$$= \lim_{h\to 0} \frac{1+h-1}{h(\sqrt{1+h}+1)} = \lim_{h\to 0} \frac{h}{h(\sqrt{1+h}+1)} = \lim_{h\to 0} \frac{1}{\sqrt{1+h}+1}$$

$$= \frac{1}{\sqrt{1+0}+1} = \frac{1}{\sqrt{1}+1} = \frac{1}{1+1} = \frac{1}{2}$$

27. Evaluate the limit.

$$\lim_{x \to -4} \frac{\frac{1}{4} + \frac{1}{x}}{4 + x} \cdot \frac{4x}{4x}$$

$$\lim_{x \to -4} \frac{\frac{4x}{4} + \frac{4x}{x}}{4x(4+x)} = \lim_{x \to -4} \frac{x+4}{4x(4+x)}$$

$$= \lim_{x \to -4} \frac{1}{4x} = -\frac{1}{16}$$

28. Evaluate the limit.

$$\lim_{x \to -1} \frac{x^2 + 2x + 1}{x^4 - 1}$$

$$\lim_{x \to -1} \frac{(x+1)^2}{(x^2-1)(x^2+1)} = \lim_{x \to -1} \frac{(x+1)^2}{(x+1)(x-1)(x^2+1)}$$

$$= \lim_{x \to -1} \frac{x+1}{(x-1)(x^2+1)} = \frac{-1+1}{(-1-1)(1+1)}$$

$$= \frac{0}{(-2)(2)} = 0$$

29. Evaluate the limit.

$$\lim_{x\to 16} \frac{4-\sqrt{x}}{16x-x^2}.$$

$$\lim_{x\to 16} \frac{(4-\sqrt{x})(4+\sqrt{x})}{(16x-x^2)(4+\sqrt{x})} = \lim_{x\to 16} \frac{4^2-(\sqrt{x})^2}{(16x-x^2)(4+\sqrt{x})}$$

$$= \lim_{x\to 16} \frac{16-x}{x(16-x)(4+\sqrt{x})} = \lim_{x\to 16} \frac{1}{x(4+\sqrt{x})}$$

$$= \frac{1}{16(4+\sqrt{16})} = \frac{1}{16(4+4)} = \frac{1}{16\cdot 8} = \frac{1}{128}$$

30. Evaluate the limit.

$$\lim_{t\to 0} \left(\frac{1}{t} - \frac{1}{t^2+t}\right)$$

$$\lim_{t\to 0} \left(\frac{1}{t} - \frac{1}{t(t+1)}\right) = \lim_{t\to 0} \left(\frac{t+1}{t(t+1)} - \frac{1}{t(t+1)}\right)$$

$$= \lim_{t\to 0} \frac{(t+1)-1}{t(t+1)} = \lim_{t\to 0} \frac{t}{t(t+1)}$$

$$= \lim_{t\to 0} \frac{1}{t+1} = \frac{1}{0+1} = 1$$

31. Find the derivative of the function using the definition of derivative.

$$f(x) = \frac{1}{2}x - \frac{1}{3}$$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{\left[\frac{1}{2}(x+h) - \frac{1}{3}\right] - \left[\frac{1}{2}x - \frac{1}{3}\right]}{h}$$

$$= \lim_{h\to 0} \frac{\frac{1}{2}x + \frac{1}{2}h - \frac{1}{3} - \frac{1}{2}x + \frac{1}{3}}{h} = \lim_{h\to 0} \frac{\frac{1}{2}h}{h} = \lim_{h\to 0} \frac{1}{2}$$

$$= \frac{1}{2}$$

32. Find the derivative of the function using the definition of derivative.

$$f(x) = 5x - 9x^2$$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{\left[5(x+h) - 9(x+h)^2\right] - \left[5x - 9x^2\right]}{h}$$

$$= \lim_{h\to 0} \frac{5x + 5h - 9(x^2 + 2xh + h^2) - 5x + 9x^2}{h}$$

$$= \lim_{h\to 0} \frac{5x + 5h - 9x^2 - 18xh - 9h^2 - 5x + 9x^2}{h}$$

$$= \lim_{h\to 0} \frac{5h - 18xh - 9h^2}{h} = \lim_{h\to 0} \frac{(5 - 18x - 9h)h}{h}$$

$$= \lim_{h\to 0} (5 - 18x - 9h) = 5 - 18x - 0 = 5 - 18x$$

33. Find the derivative of the function using the definition of derivative.

$$f(x) = \frac{1}{\sqrt{x}}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}}}{h} \cdot \frac{\sqrt{x} \cdot \sqrt{x+h}}{\sqrt{x} \cdot \sqrt{x+h}}$$

$$= \lim_{h \to 0} \frac{\sqrt{x} - \sqrt{x+h}}{h\sqrt{x} \cdot \sqrt{x+h}} \cdot \frac{\sqrt{x} + \sqrt{x+h}}{\sqrt{x} + \sqrt{x+h}}$$

$$= \lim_{h \to 0} \frac{(\sqrt{x})^2 - (\sqrt{x+h})^2}{h\sqrt{x} \cdot \sqrt{x+h}\,(\sqrt{x} + \sqrt{x+h})} = \lim_{h \to 0} \frac{x - (x+h)}{h\sqrt{x}\sqrt{x+h}(\sqrt{x} + \sqrt{x+h})}$$

$$= \lim_{h \to 0} \frac{-h}{h\sqrt{x}\sqrt{x+h}(\sqrt{x} + \sqrt{x+h})} = \lim_{h \to 0} \frac{-1}{\sqrt{x}\sqrt{x+h}(\sqrt{x} + \sqrt{x+h})}$$

$$= \frac{-1}{\sqrt{x}\sqrt{x}\,(\sqrt{x} + \sqrt{x})} = \frac{-1}{x \cdot 2\sqrt{x}} = \frac{-1}{2x^{3/2}} \quad \text{or} \quad -\frac{1}{2}x^{-3/2}$$

34. Find the derivative of the function using the definition of derivative.

$$f(x) = x^4$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{(x+h)^4 - x^4}{h}$$

$$= \lim_{h \to 0} \frac{x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4 - x^4}{h}$$

$$= \lim_{h \to 0} \frac{4x^3h + 6x^2h^2 + 4xh^3 + h^4}{h}$$

$$= \lim_{h \to 0} \left(4x^3 + 6x^2h + 4xh^2 + h^3\right)$$

$$= 4x^3 + 0 + 0 + 0$$

$$= 4x^3$$