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MAC 2241 Section U01 Test 3
Monday April 3rd
Total possible score: 20 points (2 points per page)

Question 1. Find $f'(x)$.

$$f(x) = 5x^3 + 7\sin x$$

$$f'(x) = 5 \cdot 3x^2 + 7 \cdot \cos x$$

$$= 15x^2 + 7\cos x$$

Question 2. Find $f'(x)$.

$$f(x) = \frac{3}{4x^8}$$

Easiest way: $f(x) = \frac{3}{4} \cdot \frac{1}{x^8} = \frac{3}{4} \cdot x^{-8}$

$$f'(x) = \frac{3}{4} \cdot (-8) x^{-9} = -6x^{-9} = \frac{-6}{x^9}$$

Can also use quotient rule:

$$f'(x) = \frac{(3)'(4x^8) - (3)(4x^8)'}{(4x^8)^2}$$

$$= \frac{0 \cdot 4x^8 - 3 \cdot 4 \cdot 8x^7}{4^2 x^{16}} = \frac{-3 \cdot 4 \cdot 8x^7}{4 \cdot 4x^{16}}$$

$$= \frac{-3 \cdot 8x^7}{4x^{16}} = \frac{-6}{x^9}$$

Question 3. Find $f'(x)$.

$$f(x) = \frac{x^2 + 4x + 3}{\sqrt{x}}$$

Easiest way:

$$f(x) = \frac{x^2}{\sqrt{x}} + \frac{4x}{\sqrt{x}} + \frac{3}{\sqrt{x}} = \frac{x^2}{x^{1/2}} + \frac{4x^1}{x^{1/2}} + \frac{3}{x^{1/2}}$$

$$= x^{3/2} + 4x^{1/2} + 3x^{-1/2}$$

so $f'(x) = \frac{3}{2}x^{1/2} + 4 \cdot \frac{1}{2}x^{-1/2} + 3 \cdot \frac{-1}{2}x^{-3/2}$

$= \frac{3}{2}x^{1/2} + 2x^{-1/2} - \frac{3}{2}x^{-3/2}$ which can be rewritten in various ways

Also can use quotient rule:

$$f'(x) = \frac{(2x+4)\sqrt{x} - (x^2+4x+3)\cdot\frac{1}{2}x^{-1/2}}{(\sqrt{x})^2}$$

In fact, maybe that's just as easy

Question 4. Find $f'(x)$.

$$f(x) = (x^3 + x^2 + x + 1)e^x$$

$$f'(x) = (x^3+x^2+x+1)'(e^x) + (x^3+x^2+x+1)(e^x)'$$

$$= (3x^2+2x+1+0)e^x + (x^3+x^2+x+1)e^x$$

$$= (3x^2+2x+1+x^3+x^2+x+1)e^x$$

$$= (x^3+4x^2+3x+2)e^x$$

Question 5. Find $f'(x)$ and simplify as much as possible.

$$f(x) = e^x(\sin x + \cos x)$$

$$f'(x) = (e^x)'(\sin x + \cos x) + (e^x)(\sin x + \cos x)'$$

$$= e^x(\sin x + \cos x) + e^x(\cos x - \sin x)$$

$$= e^x(\sin x + \cos x + \cos x - \sin x)$$

$$= e^x \cdot 2\cos x \quad \text{or} \quad 2e^x \cos x$$

Question 6. Find $f'(x)$ and simplify as much as possible.

$$f(x) = \frac{x^4}{1 - x^3}$$

$$f'(x) = \frac{(x^4)'(1-x^3) - (x^4)(1-x^3)'}{(1-x^3)^2}$$

$$= \frac{4x^3(1-x^3) - x^4 \cdot (-3x^2)}{(1-x^3)^2}$$

$$= \frac{4x^3 - 4x^6 + 3x^6}{(1-x^3)^2} = \frac{4x^3 - x^6}{(1-x^3)^2}$$

or $$\frac{x^3(4-x^3)}{(1-x^3)^2}$$

Question 7. Find $f'(x)$ and simplify as much as possible.

$$f(x) = \frac{\sec x}{1 + \sec x}$$

$$f'(x) = \frac{(\sec x)'(1+\sec x) - (\sec x)(1+\sec x)'}{(1+\sec x)^2}$$

$$= \frac{\sec x \tan x \cdot (1+\sec x) - \sec x \cdot \sec x \tan x}{(1+\sec x)^2}$$

$$= \frac{\sec x \tan x + \sec^2 x \tan x - \sec^2 x \tan x}{(1+\sec x)^2}$$

$$= \frac{\sec x \tan x}{(1+\sec x)^2}$$

Question 8. Find $f'(x)$.

$$f(x) = (2x^3 + 5x^2)^{100}$$

$f = u^{100}$ and $u = 2x^3 + 5x^2$

$\downarrow$ $\qquad\qquad\qquad\qquad$ $\downarrow$

$\dfrac{df}{du} = 100u^{99}$ $\qquad\qquad$ $\dfrac{du}{dx} = 6x^2 + 10x$

$$f'(x) = \frac{df}{dx} = \frac{df}{du} \cdot \frac{du}{dx} = 100u^{99} \cdot (6x^2 + 10x)$$

$$= 100(2x^3 + 5x^2)^{99} \cdot (6x^2 + 10x)$$

With experience, can "jump" to that final answer

Question 9. Find $f'(x)$.

$$f(x) = (1 + \ln x)^{1/3}$$

$f = u^{1/3}$ and $u = 1 + \ln x$

$\downarrow$ $\qquad\qquad\qquad\qquad$ $\downarrow$

$$\frac{df}{du} = \frac{1}{3}u^{-2/3} \qquad \frac{du}{dx} = \frac{1}{x}$$

$$f'(x) = \frac{df}{dx} = \frac{df}{du}\cdot\frac{du}{dx} = \frac{1}{3}u^{-2/3}\cdot\frac{1}{x}$$

$$= \frac{1}{3}(1 + \ln x)^{-2/3}\cdot\frac{1}{x}$$

With experience, can "jump" to that answer

Question 10. Find $f'(x)$.

$$f(x) = \left(\frac{x^2+1}{x^2-1}\right)^3$$

There may be more than one correct method

$$f'(x) = 3\left(\frac{x^2+1}{x^2-1}\right)^2 \cdot \left(\frac{x^2+1}{x^2-1}\right)'$$

$$= 3\left(\frac{x^2+1}{x^2-1}\right)^2 \cdot \frac{(x^2+1)'(x^2-1) - (x^2+1)(x^2-1)'}{(x^2-1)^2}$$

$$= 3\left(\frac{x^2+1}{x^2-1}\right)^2 \cdot \frac{2x(x^2-1) - (x^2+1)\cdot 2x}{(x^2-1)^2}$$

$$= 3\left(\frac{x^2+1}{x^2-1}\right)^2 \cdot \frac{2x^3 - 2x - 2x^3 - 2x}{(x^2-1)^2}$$

$$= 3\left(\frac{x^2+1}{x^2-1}\right)^2 \cdot \frac{-4x}{(x^2-1)^2} \quad \text{or} \quad \frac{-12x(x^2+1)^2}{(x^2-1)^4}$$