MAC2241 Spring 2017
Suggested problems for Test 3
(Test 3 is Monday April 3rd, in class)

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1. Find $f'(x)$.

$$f(x) = x - 3\sin x$$

$$f'(x) = 1 - 3\cos x$$

2. Find $f'(x)$.

$$f(x) = 3e^x + \frac{4}{x^{1/3}} = 3e^x + 4x^{-1/3}$$

$$f'(x) = 3e^x + 4 \cdot \frac{-1}{3} x^{-4/3} \quad \text{or} \quad 3e^x - \frac{4}{3x^{4/3}}$$

3. Find $f'(x)$.

$$f(x) = \sqrt{x}(x-1)$$

$$f(x) = x^{1/2} \cdot (x-1) = x^{3/2} - x^{1/2}$$

$$f'(x) = \frac{3}{2} x^{1/2} - \frac{1}{2} x^{-1/2} \quad \text{or} \quad \frac{3\sqrt{x}}{2} - \frac{1}{2\sqrt{x}}$$

which can also be written as $\frac{3x}{2\sqrt{x}} - \frac{1}{2\sqrt{x}} = \frac{3x-1}{2\sqrt{x}}$

4. Find $f'(x)$.

$$f(x) = \frac{x^2 - 3x + 1}{x^2} = \frac{x^2}{x^2} - \frac{3x}{x^2} + \frac{1}{x^2}$$

$$f(x) = 1 - 3x^{-1} + x^{-2}$$

$$f'(x) = +3x^{-2} - 2x^{-3} \quad \text{or} \quad \frac{3}{x^2} - \frac{2}{x^3} \quad \text{or} \quad \frac{3x-2}{x^3}$$

5. Find $f'(x)$.

$$f(x) = (x^3 + 2x)e^x$$

$$f'(x) = (x^3+2x)'e^x + (x^3+2x)(e^x)'$$

$$= (3x^2+2)e^x + (x^3+2x)e^x$$

$$\text{or} \quad (x^3+3x^2+2x+2)e^x$$

6. Find $f'(x)$.

$$f(x) = e^x \cos x$$

$$f'(x) = (e^x)'\cos x + e^x(\cos x)'$$

$$= e^x \cos x - e^x \sin x$$

$$\text{or} \quad e^x(\cos x - \sin x)$$

7. Find $f'(x)$.

$$f(x) = \frac{x^3}{1-x^2}$$

$$f'(x) = \frac{(x^3)'(1-x^2) - x^3(1-x^2)'}{(1-x^2)^2}$$

$$= \frac{3x^2(1-x^2) - x^3(-2x)}{(1-x^2)^2}$$

Could stop there, but it's good practice to simplify.

$$\frac{3x^2 - 3x^4 + 2x^4}{(1-x^2)^2} = \frac{3x^2 - x^4}{(1-x^2)^2}$$

8. Find $f'(x)$.

$$f(x) = \frac{1 - xe^x}{x + e^x}$$

$$f'(x) = \frac{(1-xe^x)'(x+e^x) - (1-xe^x)(x+e^x)'}{(x+e^x)^2}$$

$$= \frac{(-e^x - xe^x)(x+e^x) - (1-xe^x)(1+e^x)}{(x+e^x)^2}$$

Could stop there, or could rewrite

$$\frac{-xe^x - e^{2x} - x^2e^x - xe^{2x} - 1 - e^x + xe^x + xe^{2x}}{(x+e^x)^2}$$

$$\frac{-e^{2x} - (x^2+1)e^x - 1}{(x+e^x)^2}$$

9. Find $f'(x)$.

$$f(x) = \frac{1 - \sec x}{\tan x}$$

$$f'(x) = \frac{(1-\sec x)' \tan x - (1-\sec x)(\tan x)'}{\tan^2 x}$$

$$= \frac{-\sec x \tan x \cdot \tan x - (1-\sec x)\sec^2 x}{\tan^2 x} = \frac{-\sec x \tan^2 x - \sec^2 x + \sec^3 x}{\tan^2 x}$$

which can be rewritten

$$\frac{-\sec x(\sec^2 x - 1) - \sec^2 x + \sec^3 x}{\tan^2 x}$$

$$\frac{\sec x - \sec^2 x}{\tan^2 x}$$

10. Find $f'(x)$.

$$f(x) = (x^4 + 3x^2 - 2)^5$$

$$f'(x) = 5(x^4 + 3x^2 - 2)^4 \cdot (x^4 + 3x^2 - 2)'$$

$$= 5(x^4 + 3x^2 - 2)^4 \cdot (4x^3 + 6x)$$

11. Find $f'(x)$.

$$f(x) = \sqrt{1 - 2x} = (1-2x)^{1/2}$$

$$f'(x) = \frac{1}{2}(1-2x)^{-1/2} \cdot (1-2x)'$$

$$= \frac{1}{2}(1-2x)^{-1/2} \cdot (-2)$$

or $\dfrac{-1}{\sqrt{1-2x}}$

12. Find $f'(x)$.

$$f(x) = (2x-5)^4(8x^2-5)^{-3}$$

One method: $f'(x) = \left((2x-5)^4\right)'(8x^2-5)^{-3} + (2x-5)^4\left((8x^2-5)^{-3}\right)'$

$= 4(2x-5)^3 \cdot 2 \cdot (8x^2-5)^{-3} + (2x-5)^4 \cdot (-3)(8x^2-5)^{-4} \cdot 16x$

Another method: $y = (2x-5)^4(8x^2-5)^{-3}$

$\ln y = 4\ln(2x-5) - 3\ln(8x^2-5)$

$\frac{1}{y}y' = 4 \cdot \frac{1}{2x-5} \cdot 2 - 3\frac{1}{8x^2-5} \cdot 16x \Rightarrow y' = y \cdot \left(\frac{8}{2x-5} - \frac{48x}{8x^2-5}\right)$

13. Find $f'(x)$.

$$f(x) = e^{x\cos x}$$

$f'(x) = e^{x\cos x} \cdot (x\cos x)'$

$= e^{x\cos x} \cdot \left((x)'\cos x + x(\cos x)'\right)$

$= e^{x\cos x} \cdot (1\cos x - x\sin x)$

$f = 10^u$ and $u = 1 - x^2$

14. Find $f'(x)$.

$$f(x) = 10^{1-x^2}$$

$$\frac{df}{du} = 10^u \ln u$$

$$f'(x) = 10^{1-x^2} \cdot \ln 10 \cdot (1-x^2)'$$

$$= 10^{1-x^2} \cdot \ln 10 \cdot (-2x)$$

15. Find $f'(x)$.

$$f(x) = \sec^2 x + \tan^2 x = (\sec x)^2 + (\tan x)^2$$

$$f'(x) = 2\sec x \cdot (\sec x)' + 2\tan x \cdot (\tan x)'$$

$$= 2\sec x \cdot \sec x \tan x + 2\tan x \cdot \sec^2 x$$

$$= 2\sec^2 x \tan x + 2\sec^2 x \tan x$$

$$= 4\sec^2 \tan x$$

Can probably be written in other ways as well

16. Find $f'(x)$.

$$f(x) = \frac{x}{\sqrt{x^2+1}} = \frac{x}{(x^2+1)^{1/2}}$$

$$f'(x) = \frac{1(x^2+1)^{1/2} - x\cdot\frac{1}{2}(x^2+1)^{-1/2}\cdot 2x}{x^2+1} = \frac{\sqrt{x^2+1} - \frac{x^2}{\sqrt{x^2+1}}}{x^2+1}$$

Can multiply by $\frac{\sqrt{x^2+1}}{\sqrt{x^2+1}}$ to get $\frac{x^2+1-x^2}{(x^2+1)^{3/2}} = \frac{1}{(x^2+1)^{3/2}}$

17. Find $f'(x)$.

$$f(x) = x\ln x - x$$

$$f'(x) = 1\ln x + x\cdot\frac{1}{x} - 1 = \ln x + 1 - 1$$

$$= \ln x$$

18. Find $f'(x)$.

$$f(x) = \sin(\ln x)$$

$$f'(x) = \cos(\ln x)\cdot(\ln x)'$$

$$= \cos(\ln x)\cdot\frac{1}{x} = \frac{\cos(\ln x)}{x}$$

19. Find $f'(x)$.

$$f(x) = (\ln x)^{1/5}$$

$$f'(x) = \frac{1}{5}(\ln x)^{-4/5} \cdot \frac{1}{x}$$

20. Find $f'(x)$.

$$f(x) = \left(\ln(1 + e^x)\right)^2$$

$$f'(x) = 2(\ln(1+e^x)) \cdot (\ln(1+e^x))'$$

$$= 2\ln(1+e^x) \cdot \frac{1}{1+e^x} \cdot (1+e^x)'$$

$$= 2\ln(1+e^x) \cdot \frac{1}{1+e^x} \cdot e^x$$

21. Find $f'(x)$.

$$f(x) = \sin x \ln(5x)$$

$$f'(x) = (\sin x)' \ln(5x) + \sin x\,(\ln(5x))'$$

$$= \cos x \ln(5x) + \sin x \cdot \frac{1}{5x} \cdot 5$$

$$= \cos x \ln(5x) + \frac{\sin x}{x}$$