MAC2311 Fall 2017
Suggested problems for final exam.
The final exam is **cumulative**.
You should **also** practice the
suggested problems for Tests 1 through 3.

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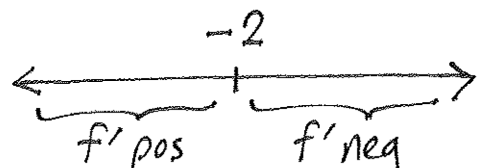
1. Find the intervals on which the function is increasing, decreasing, concave up, or concave down.

$$f(x) = 5 - 4x - x^2$$

$f'(x) = -4 - 2x$

$f''(x) = -2$

$f' = 0$? $-4 - 2x = 0 \Rightarrow -2x = 4 \Rightarrow x = -2.$



f'' is always negative.

f is increasing if $x < -2$, decreasing if $x > -2$, and concave down for all x.

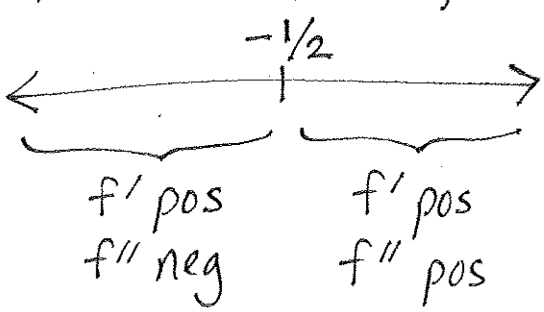
2. Find the intervals on which the function is increasing, decreasing, concave up, or concave down.

$$f(x) = (2x + 1)^3$$

$f'(x) = 3(2x+1)^2 \cdot 2 = 6(2x+1)^2$

$f''(x) = 6 \cdot 2(2x+1) \cdot 2 = 24(2x+1)$

$f' = 0$ if $2x = -1$, i.e. if $x = -1/2$, and $f'' = 0$ if $x = -1/2$



f is increasing for all x

f is concave up if $x > -1/2$

f is concave down if $x < -1/2$

3. Find the intervals on which the function is increasing, decreasing, concave up, or concave down.

$$f(x) = 5 + 12x - x^3$$

$$f'(x) = 12 - 3x^2$$

$$f''(x) = -6x$$

$f' = 0$? $12 - 3x^2 = 0$

$12 = 3x^2$

$4 = x^2 \Rightarrow x = -2$ or $+2$

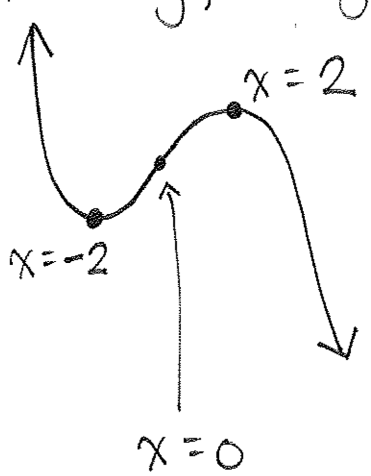
$f'' = 0$? $-6x = 0 \Rightarrow x = 0$

-2 $\quad$ 2

f' neg	f' pos	f' neg
f decr	f incr	f decr

f'' pos	f'' neg
f concave up	f concave down

So by the way, the graph of f would look something like



4. Find the intervals on which the function is increasing, decreasing, concave up, or concave down.

$$f(x) = \frac{x}{x^2+4}$$

$$f'(x) = \frac{(x)'(x^2+4) - x(x^2+4)'}{(x^2+4)^2} = \frac{1(x^2+4) - x\cdot 2x}{(x^2+4)^2} = \frac{4-x^2}{(x^2+4)^2}$$

$$f''(x) = \frac{(4-x^2)'(x^2+4)^2 - (4-x^2)((x^2+4)^2)'}{((x^2+4)^2)^2}$$

$$= \frac{-2x(x^2+4)^2 - (4-x^2)\,2(x^2+4)\cdot 2x}{(x^2+4)^4}$$

$$= \frac{-2x(x^2+4)\left[(x^2+4) + 2(4-x^2)\right]}{(x^2+4)^4}$$

$$= \frac{-2x(x^2+4+8-2x^2)}{(x^2+4)^3} = \frac{-2x(12-x^2)}{(x^2+4)^3}$$

Notice denominators of f' and f'' are always positive

f' changes sign at $x=-2$ and $x=2$

f'' changes sign at $x=-\sqrt{12}$, $x=0$, and $x=\sqrt{12}$

f is decreasing on $(-\infty,-2)$, increasing on $(-2,2)$, decreasing on $(2,\infty)$

f is conc. down on $(-\infty,-\sqrt{12})$, conc. up on $(-\sqrt{12},0)$, conc. down on $(0,\sqrt{12})$, conc. up on $(\sqrt{12},\infty)$

5. Find the intervals on which the function is increasing, decreasing, concave up, or concave down.

$$f(x) = x^{4/3} - x^{1/3}$$

$$f'(x) = \frac{4}{3}x^{1/3} - \frac{1}{3}x^{-2/3} = \frac{4x^{1/3}}{3} - \frac{1}{3x^{2/3}}$$

$$= \frac{4x}{3x^{2/3}} - \frac{1}{3x^{2/3}} = \frac{4x-1}{3x^{2/3}}$$

$$f''(x) = \frac{4}{3}\cdot\frac{1}{3}x^{-2/3} - \frac{1}{3}\cdot\frac{-2}{3}x^{-5/3}$$

$$= \frac{4}{9x^{2/3}} + \frac{2}{9x^{5/3}} = \frac{4x}{9x^{5/3}} + \frac{2}{9x^{5/3}} = \frac{4x+2}{9x^{5/3}}$$

$f'(x) = \dfrac{4x-1}{3(x^{1/3})^2}$ Bottom is never negative
Top changes sign at $x = \frac{1}{4}$

$f''(x) = \dfrac{4x+2}{9(x^{1/3})^5}$ Top changes sign at $x = -\frac{1}{2}$
Bottom changes sign at $x = 0$

f is decreasing on the interval $(-\infty, \frac{1}{4})$ and increasing on $(\frac{1}{4}, \infty)$

f is concave up on $(-\infty, -\frac{1}{2})$, concave down on $(-\frac{1}{2}, 0)$
and concave up on $(0, \infty)$

6. Find all the critical points of the function.

$$f(x) = 4x^4 - 16x^2 + 17$$

$$f'(x) = 16x^3 - 32x$$
$$= 16x(x^2 - 2)$$

Critical points when $x = 0$ or $x^2 = 2$

i.e. $x = 0$, $x = -\sqrt{2}$, $x = \sqrt{2}$

7. Find all the critical points of the function.

$$f(x) = 3x^4 + 12x$$

$$f'(x) = 12x^3 + 12$$
$$= 12(x^3 + 1)$$

Critical points when $x^3 + 1 = 0$

$$x^3 = -1$$
$$x = -1$$

8. Find all the critical points of the function.

$$f(x) = \frac{x+1}{x^2+3}$$

$$f'(x) = \frac{(x+1)'(x^2+3)-(x+1)(x^2+3)'}{(x^2+3)^2} = \frac{1(x^2+3)-(x+1)\cdot 2x}{(x^2+3)^2}$$

$$= \frac{x^2+3-2x^2-2x}{(x^2+3)^2} = \frac{-x^2-2x+3}{(x^2+3)^2} = \frac{(-x+1)(x+3)}{(x^2+3)^2}$$

Top is 0 when $x=1$ or $x=-3$

Bottom is never 0. Critical points: $x=1$, $x=-3$

9. Find all the critical points of the function.

$$f(x) = (x^2-25)^{1/3}$$

$$f'(x) = \frac{1}{3}(x^2-25)^{-2/3}\cdot 2x = \frac{2x}{3(x^2-25)^{2/3}}$$

Top is 0 when $x=0$

Bottom is 0 when $x^2-25=0$ i.e. $x^2=25$ i.e. $x=5$ or -5

Critical points: $x=-5$, $x=0$, $x=5$

10. Find all the relative extrema of the function, and classify each of them as a minimum or a maximum.

$$f(x) = x^4 - 4x^3 + 4x^2$$

$$f'(x) = 4x^3 - 12x^2 + 8x$$
$$= 4x(x^2 - 3x + 2)$$
$$= 4x(x-1)(x-2)$$

Critical numbers: $x = 0$, $x = 1$, $x = 2$

	0		1		2	
$4x$	$- - -$ 0	$+ + + +$	$+ + + +$	$+ + + +$	$+ + + +$	$+ +$
$x-1$	$- - -$	$- - -$	0	$+ + + +$	$+ + + +$	$+ +$
$x-2$	$- - -$	$- - -$	$- - -$	$- - -$	0	$+ + + +$

	0		1		2	
f'	$- - - -$	0	$+ + + +$	0	$- - - -$	0	$+ + + +$
	f decr	min	f incr	max	f decr	min	f incr

11. Find all the relative extrema of the function, and classify each of them as a minimum or a maximum.

$$f(x) = 2x + 3x^{2/3}$$

$$f'(x) = 2 + 3 \cdot \frac{2}{3} x^{-1/3} = 2 + \frac{2}{x^{1/3}}$$

$$f'(x) = \frac{2x^{1/3}}{x^{1/3}} + \frac{2}{x^{1/3}} = \frac{2x^{1/3} + 2}{x^{1/3}} = \frac{2(x^{1/3}+1)}{x^{1/3}}$$

Critical numbers? Top is 0 when $x^{1/3} = -1$ i.e. $x = -1$

Bottom is 0 when $x = 0$

		-1		0	
$x^{1/3}+1$	$- - - -$	0	$+ + + + +$	$+ + + +$	$+ + +$
$x^{1/3}$	$- - - -$	$- - - -$	$- - - -$	0	$+ + + +$
f'	$+ + + + +$	0	$- - - -$	undef	$+ + + +$

f has a max at $x = -1$

f has a min at $x = 0$

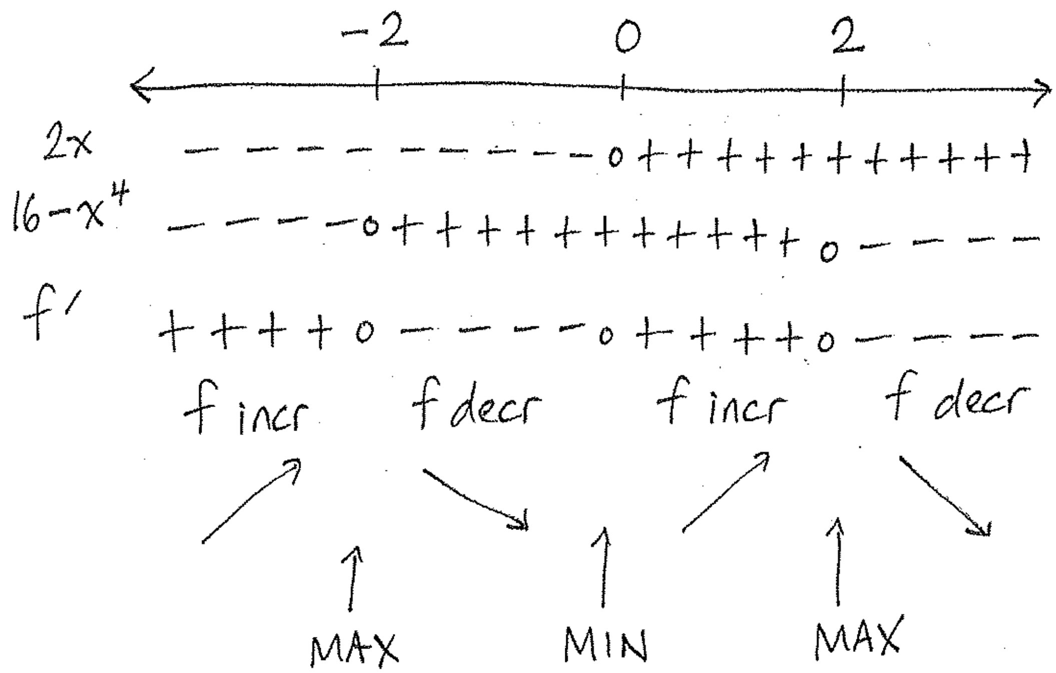
12. Find all the relative extrema of the function, and classify each of them as a minimum or a maximum.

$$f(x) = \frac{x^2}{x^4+16}$$

$$f'(x) = \frac{(2x)(x^4+16)-(x^2)(4x^3)}{(x^4+16)^2} = \frac{2x^5+32x-4x^5}{(x^4+16)^2}$$

$$= \frac{32x-2x^5}{(x^4+16)^2} = \frac{2x(16-x^4)}{(x^4+16)^2}$$

Bottom is never 0
Top is 0 if $x=0$
or $x^4=16$
$x=\pm 2$



13. Find all the relative extrema of the function, and classify each of them as a minimum or a maximum.

$$f(x) = \ln(2 + x^2)$$

$$f'(x) = \frac{1}{2+x^2} \cdot (2+x^2)' = \frac{1}{2+x^2} \cdot 2x = \frac{2x}{2+x^2}$$

Top is 0 when $x=0$

Bottom is 0 never

$x=0$

$f' = \dfrac{2x}{2+x^2}$

f' is negative

f is decreasing

f' is positive

f is increasing

Minimum at $x=0$

14. Draw a graph of the function. Label all the critical points, inflection points, and asymptotes.

$$f(x) = \frac{x-3}{4-x}$$

Vertical asymptote at $x=4$.

Note that, for example, $f(3.999) = \frac{\text{positive near } 1}{\text{positive near } 0}$ = extreme positive

and $f(4.001) = \frac{\text{positive near } 1}{\text{negative near } 0}$ = extreme negative.

Also note $f(x) = \frac{1x + (\text{smaller})}{-1x + (\text{smaller})}$ so if $x \to \pm\infty$, then $f(x) \to \frac{1}{-1} = -1$.

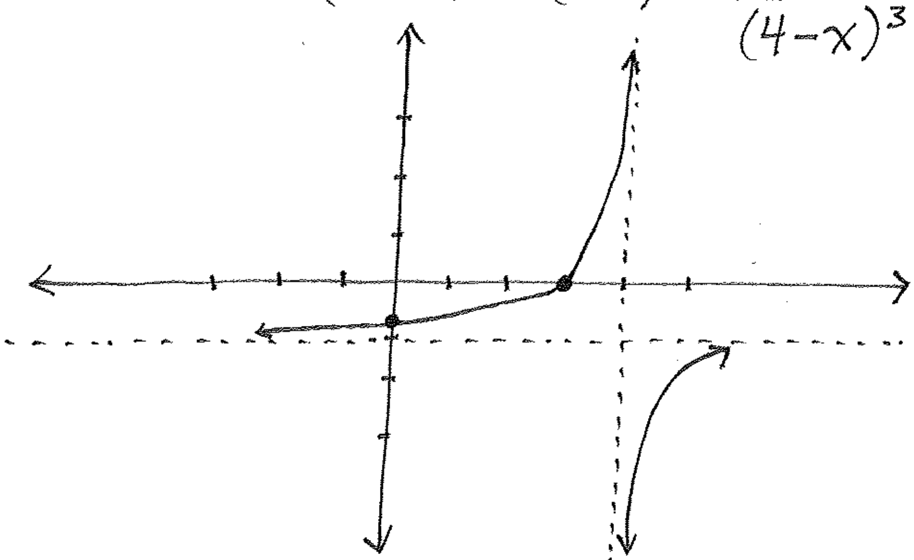
So $y=-1$ is a horizontal asymptote.

Intercepts: $f(0) = -\frac{3}{4}$ $\quad f(3) = 0$.

Derivative: $f' = \frac{1(4-x)-(x-3)(-1)}{(4-x)^2} = \frac{4-x+x-3}{(4-x)^2} = \frac{1}{(4-x)^2}$

so f' is never negative. $f' = (4-x)^{-2}$ so then

$f'' = -2(4-x)^{-3}\cdot(-1) = \frac{2}{(4-x)^3}$ has same sign as $4-x$.



Always increasing
Left part concave up
Right part concave down

15. Draw a graph of the function. Label all the critical points, inflection points, and asymptotes.

$$f(x) = \frac{x}{x^2 - 4} = \frac{x}{(x+2)(x-2)}$$

Vertical asymptotes at $x = -2$, $x = 2$

$f(x) = \frac{x + \text{smaller}}{x^2 + \text{smaller}} = \frac{\deg 1}{\deg 2}$, so if $x \to \pm\infty$ then $f(x) \to 0$

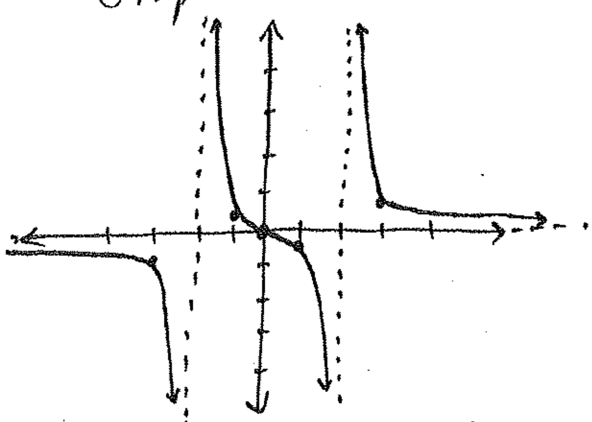
$y = 0$ is horizontal asymptote

$f(0) = 0$ is only intercept

$$f'(x) = \frac{(x)'(x^2-4) - (x)(x^2-4)'}{(x^2-4)^2} = \frac{1(x^2-4) - x \cdot 2x}{(x^2-4)^2}$$

$$= \frac{x^2 - 4 - 2x^2}{(x^2-4)^2} = \frac{-x^2-4}{(x^2-4)^2} = \frac{-(x^2+4)}{(x^2-4)^2}$$ always negative

Graph so far:



$f(2.01) = \frac{\text{pos}}{\text{pos}}$ $\quad f(1.99) = \frac{\text{pos}}{\text{neg}}$

$f(-2.01) = \frac{\text{neg}}{\text{pos}}$ $\quad f(-1.99) = \frac{\text{neg}}{\text{neg}}$

I just guessed when joining points with a curve, but to find inflection points, we need f''. It's hard to fit all that on one page. It would start with

$$f''(x) = -\frac{(x^2+4)'(x^2-4)^2 - (x^2+4)((x^2-4)^2)'}{(x^2-4)^4}$$

$$= -\frac{2x(x^2-4)^2 - (x^2+4) \cdot 2(x^2-4) \cdot 2x}{(x^2-4)^4}$$

See next page

$$f''(x) = \frac{-2x(x^2-4)\left[(x^2-4)-(x^2+4)\cdot 2\right]}{(x^2-4)^4}$$

$$= \frac{-2x(x^2-4-2x^2-8)}{(x^2-4)^3}$$

$$= \frac{-2x(-x^2-12)}{(x^2-4)^3}$$

$$= \frac{2x(x^2+12)}{(x^2-4)^3}$$

Top has same sign as x
Bottom has same sign as x^2-4

Concavity will change at $x=-2$, $x=0$, $x=2$
(as my graph correctly guessed)
$x=0$ is the only inflection point.

16. Find the absolute maximum and absolute minimum of the function

$$f(x) = (x^2 + x)^{2/3}$$

on the interval $[-2, 3]$.

$$f'(x) = \frac{2}{3}(x^2+x)^{-1/3} \cdot (2x+1) = \frac{2(2x+1)}{3(x^2+x)^{1/3}}$$

$f' = 0$ when $x = -\frac{1}{2}$ $\quad$ f' undefined when $x = 0$ or $x = -1$

Check critical points and endpoints.

$f(-2) = (4-2)^{2/3} = 2^{2/3}$ $\qquad$ $f(0) = (0+0)^{2/3} = 0^{2/3} = 0$

$f(-1) = (1-1)^{2/3} = 0^{2/3} = 0$ $\qquad$ $f(3) = (9+3)^{2/3} = 12^{2/3}$

$f(-\frac{1}{2}) = (\frac{1}{4} - \frac{1}{2})^{2/3} = (-\frac{1}{4})^{2/3}$ $\qquad$ Min is 0, max is $12^{2/3}$

17. Find the absolute maximum and absolute minimum of the function

$$f(x) = \frac{x-2}{x+1}$$

on the interval $(-1, 5]$.

$$f'(x) = \frac{(x-2)'(x+1) - (x-2)(x+1)'}{(x+1)^2} = \frac{1(x+1) - (x-2)\cdot 1}{(x+1)^2}$$

$$= \frac{x+1-x+2}{(x+1)^2} = \frac{3}{(x+1)^2}.$$

f' is never 0.
f' is undefined when $x = -1$.

Check critical points and ends of domain.

$$f(5) = \frac{5-2}{5+1} = \frac{3}{6} = \frac{1}{2}$$

$f(\text{near } -1) = \text{e.g. } f(-0.999) = \dfrac{\text{near } -3}{\text{positive near } 0} = $ extreme negative number

Note: x must be slightly <u>more</u> than -1 e.g. $x = -0.999$

Max is $\frac{1}{2}$. There is no min.

18 Find the absolute maximum and absolute minimum of the function

$$f(x) = \frac{\ln x}{x}$$

on the interval $[1, e^2]$. $f'(x) = \dfrac{(\ln x)'x - (\ln x)(x)'}{x^2}$

$= \dfrac{\frac{1}{x}\cdot x - (\ln x)\cdot 1}{x^2} = \dfrac{1-\ln x}{x^2}$ f' undefined when $x=0$ (not in domain)

$f'=0$ when $\ln x = 1$

$x = e$

Check $f(1), f(e), f(e^2)$.

MORE than $\frac{1}{3}$ LESS than $\frac{1}{3}$

$f(1) = \frac{\ln 1}{1} = \frac{0}{1} = 0$ $f(e) = \frac{\ln e}{e} = \frac{1}{e}$ $f(e^2) = \frac{\ln(e^2)}{e^2} = \frac{2}{e^2}$

19. Find a number in the closed interval $[\frac{1}{2}, \frac{3}{2}]$ such that the sum of the number and its reciprocal is
a. as small as possible
b. as large as possible.

Min is 0

Max is $\frac{1}{e} > \frac{1}{3}$

Let x be the number in $[\frac{1}{2}, \frac{3}{2}]$. Its reciprocal is $\frac{1}{x}$.

Want to minimize or maximize $x + \frac{1}{x} = x + x^{-1}$.

If $f(x) = x + x^{-1}$ then $f'(x) = 1 - x^{-2} = 1 - \frac{1}{x^2}$

$= \frac{x^2-1}{x^2}$. f' undefined when $x=0$ (not in domain)

$f'=0$ when $x=-1$ (not in domain) or $x=1$.

Check $x = \frac{1}{2}, 1, \frac{3}{2}$. $f(\frac{1}{2}) = \frac{1}{2} + 2 = \frac{1}{2} + \frac{4}{2} = \frac{5}{2} = 2.5$

$f(1) = 1+1 = 2$ $f(\frac{3}{2}) = \frac{3}{2} + \frac{2}{3} = \frac{9}{6} + \frac{4}{6} = \frac{13}{6} = 2 + \frac{1}{6}$

Max is 2.5, min is 2. To minimize, choose $x=1$.

To maximize, choose $x = \frac{1}{2}$.

20. How should two nonnegative numbers be chosen so that their sum is 1 and the sum of their squares is
a. as large as possible
b. as small as possible?

We could call the two numbers x and y.

Given: Their sum is 1. $x+y=1$. $y=1-x$

Given: They are nonnegative. $x \geq 0$. $y \geq 0$

$1-x \geq 0$

$1 \geq x \Rightarrow x \leq 1$

Want to minimize or maximize the sum of their squares

i.e. $x^2+y^2 = x^2+(1-x)^2$.

Let $f(x) = x^2+(1-x)^2 = x^2+1-2x+x^2$

$= 2x^2-2x+1$, so $f'(x) = 4x-2$.

Critical points? f' is never undefined. $f'=0$? $4x-2=0$

$4x = 2$

$x = \frac{2}{4} = \frac{1}{2}$

Check $x=0$, $x=\frac{1}{2}$, $x=1$.

$f(0) = 0^2+(1-0)^2 = 0+1 = 1$

$f(\frac{1}{2}) = (\frac{1}{2})^2+(1-\frac{1}{2})^2 = \frac{1}{4}+\frac{1}{4} = \frac{1}{2}$

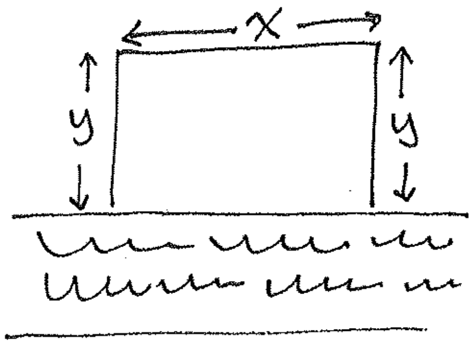
$f(1) = 1^2+(1-1)^2 = 1+0 = 1$

To maximize x^2+y^2, choose $x,y = 0,1$.

To minimize x^2+y^2, choose $x=y=\frac{1}{2}$.

21. A rectangular field is to be bounded by a fence on three sides and by a straight stream on the fourth side. Find the dimensions of the field with maximum area that can be enclosed using 1000 ft of fence.

DRAW A PICTURE and MAKE UP NAMES.



Total length of fence $= x + 2y$

WANT to maximize the area

Area $= xy$

GIVEN: Total length of fence $= 1000$

$$x + 2y = 1000$$
$$2y = 1000 - x$$
$$y = 500 - \tfrac{1}{2}x \Rightarrow \text{Area} = xy = x\left(500 - \tfrac{1}{2}x\right)$$

Area $= 500x - \frac{1}{2}x^2$. Call this $f(x)$. We want to maximize $f(x)$.

$f'(x) = 500 - x$. Critical numbers? f' is never undefined. $f' = 0$ if $x = 500$.

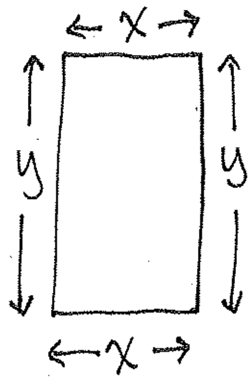
Check the critical number $x = 500$ and the ends of the domain which are the extreme cases: $x = 0$ or $x = 1000$.

$$f(0) = 0 \cdot \left(500 - \tfrac{1}{2} \cdot 0\right) = 0$$
$$f(500) = 500 \cdot \left(500 - \tfrac{1}{2} \cdot 500\right) = 500 \cdot 250 = 125{,}000$$
$$f(1000) = 1000 \cdot \left(500 - \tfrac{1}{2} \cdot 1000\right) = 0$$

The field should be 500 ft by 250 ft

22. A rectangular plot of land is to be fenced in using two kinds of fencing. Two opposite sides will use heavy-duty fencing selling for \$3 a foot, while the remaining two sides will use standard fencing selling for \$2 a foot. What are the dimensions of the rectangular plot of greatest area that can be fenced in for a cost of \$6000?

DRAW A PICTURE and MAKE UP NAMES



Suppose the sides of length x are heavy-duty fencing (\$3 per foot)
Suppose the sides of length y are standard fencing (\$2 per foot)

Want to MAXIMIZE the area. Area $= xy$

Total cost $= \$6000$.

Total length of heavy-duty fence $= 2x$ feet.
Total cost of heavy-duty fence $= (2x \text{ feet}) \cdot (3 \text{ dollars per foot})$
$= 6x$ dollars.

Similarly, total cost of standard fence $= 2y \cdot 2 = 4y$.

So, $6x + 4y = 6000 \Rightarrow 3x + 2y = 3000$

$$2y = 3000 - 3x$$
$$y = 1500 - \frac{3}{2}x$$

$$\text{Area} = xy = x\left(1500 - \frac{3}{2}x\right) = 1500x - \frac{3}{2}x^2$$

Call this $f(x)$.
Want to maximize $f(x)$.
NEXT PAGE

$f(x) = 1500x - \frac{3}{2}x^2$

$f'(x) = 1500 - 3x$

Critical numbers? f' is never undefined

$f' = 0$ when $x = 500$

Check $x = 500$ and the ends of the domain.

WHAT ARE the ends of the domain?

What are restrictions on x or y? Total cost $= 6000$

Cost of heavy-duty fence is $6x$, so $6x \le 6000$

$x \le 1000$

Also $x \ge 0$ because x is a length.

Check $x = 0$, $x = 500$, $x = 1000$.

$f(0) = 0 \cdot \left(1500 - \frac{3}{2} \cdot 0\right) = 0$

$f(500) = 500 \cdot \left(1500 - \frac{3}{2} \cdot 500\right)$

$= 500 \cdot (1500 - 750) = 500 \cdot 750$

$f(1000) = 1000 \cdot \left(1500 - \frac{3}{2} \cdot 1000\right)$

$= 1000 \cdot (1500 - 1500) = 0$

Area is maximized when $x = 500$ and $y = 750$.

23. Evaluate the integral.

$$\int (x^{-3} - 3x^{1/4} + 8x^2)\,dx$$

$$= \int x^{-3}\,dx - \int 3x^{1/4}\,dx + \int 8x^2\,dx$$

$$= \int x^{-3}\,dx - 3\int x^{1/4}\,dx + 8\int x^2\,dx$$

$$= \frac{x^{-3+1}}{-3+1} - 3\cdot\frac{x^{\frac{1}{4}+1}}{\frac{1}{4}+1} + 8\cdot\frac{x^3}{3} + C$$

$$\boxed{\frac{x^{-2}}{-2} - 3\cdot\frac{x^{5/4}}{5/4} + \frac{8x^3}{3} + C}$$

or

$$\boxed{-\frac{1}{2x^2} - 3\cdot\frac{4}{5}x^{5/4} + \frac{8x^3}{3} + C}$$

24. Evaluate the integral.

$$\int x(1+x^3)\,dx = \int (x + x^4)\,dx$$

$$= \frac{x^2}{2} + \frac{x^5}{5} + C$$

25. Evaluate the integral.

$$\int \frac{x^5 + 2x^2 - 1}{x^4}\,dx$$

$$\int \left(\frac{x^5}{x^4} + \frac{2x^2}{x^4} - \frac{1}{x^4} \right) dx = \int (x + 2x^{-2} - x^{-4})\,dx$$

$$= \frac{x^2}{2} + 2 \cdot \frac{x^{-1}}{-1} - \frac{x^{-3}}{-3} + C$$

or $\frac{x^2}{2} - \frac{2}{x} + \frac{1}{x^3} + C$

26. Evaluate the integral.

$$\int (3 \sin x - 2 \sec^2 x)\,dx$$

$$= \int 3 \sin x\,dx - \int 2 \sec^2 x\,dx$$

$$= 3 \int \sin x\,dx - 2 \int \sec^2 x\,dx$$

$$= 3(-\cos x) - 2 \tan x + C$$

or $-3 \cos x - 2 \tan x + C$

27. Evaluate the integral.

$$\int \sec x(\sec x + \tan x)\, dx$$

$$= \int \left(\sec^2 x + \sec x \tan x\right) dx$$

$$= \int \sec^2 x \, dx + \int \sec x \tan x \, dx$$

$$= \tan x + \sec x + C$$

28. Evaluate the integral.

$$\int 2x(x^2+1)^{23}\, dx$$

Try substitution. Try $u = x^2 + 1$

$$\Rightarrow \frac{du}{dx} = 2x \Rightarrow du = 2x\, dx$$

$$\text{Integral} = \int \underbrace{(x^2+1)^{23}}_{u^{23}} \cdot \underbrace{2x\, dx}_{du} = \int u^{23}\, du$$

$$= \frac{u^{24}}{24} + C = \frac{(x^2+1)^{24}}{24} + C$$

29. Evaluate the integral.

$$\int \cos^3 x \sin x \, dx$$

Try substitution. Try $u = \cos x \Rightarrow \frac{du}{dx} = -\sin x$

$\Rightarrow du = -\sin x \, dx$

$\Rightarrow -du = \sin x \, dx$ Integral $= \int \underbrace{(\cos x)^3}_{u^3} \underbrace{\sin x \, dx}_{-du}$

$$= \int -u^3 \, du = -\frac{u^4}{4} + C = -\frac{\cos^4 x}{4} + C$$

30. Evaluate the integral.

$$\int \sec^2(4x+1) \, dx$$

Try substitution. Try $u = 4x+1 \Rightarrow \frac{du}{dx} = 4$

$\Rightarrow du = 4dx \Rightarrow \frac{1}{4} du = dx$

Integral $= \int \underbrace{\sec^2(4x+1)}_{\sec^2 u} \underbrace{dx}_{\frac{1}{4}du} = \int \frac{1}{4} \sec^2 u \, du$

$$= \frac{1}{4} \int \sec^2 u \, du = \frac{1}{4} \tan u + C$$

$$= \frac{1}{4} \tan(4x+1) + C$$

31. Evaluate the integral.

$$\int \frac{3x}{\sqrt{4x^2+5}}\,dx$$

Try $u = 4x^2+5 \Rightarrow \frac{du}{dx} = 8x \Rightarrow du = 8x\,dx$

$\Rightarrow \frac{1}{8}du = x\,dx$. Integral $= \int 3 \cdot \underbrace{(4x^2+5)^{-1/2}}_{u^{-1/2}} \cdot \underbrace{x\,dx}_{\frac{1}{8}du}$

$= \int \frac{3}{8} u^{-1/2}\,du = \frac{3}{8} \cdot \frac{u^{1/2}}{1/2} + C = \frac{3}{8} \cdot \frac{2}{1} \cdot u^{1/2} + C$

$= \frac{3}{4}\sqrt{u} + C$

$= \frac{3}{4}\sqrt{4x^2+5} + C.$

32. Evaluate the integral.

$$\int (4x-3)^9\,dx$$

Sub $u = 4x-3$

$\frac{du}{dx} = 4 \Rightarrow du = 4\,dx$

$\frac{1}{4}du = dx$

$\int (4x-3)^9\,dx = \int u^9 \cdot \frac{1}{4}\,du = \frac{1}{4} \cdot \frac{u^{10}}{10} + C$

$= \frac{(4x-3)^{10}}{40} + C.$

33 . Evaluate the integral.

$$\int \sec 4x \tan 4x \, dx$$

Try $u = 4x \Rightarrow \frac{du}{dx} = 4 \Rightarrow du = 4dx$

$\frac{1}{4} du = dx.$

$$\int \sec 4x \tan 4x \, dx = \int \sec u \tan u \, \frac{1}{4} du$$

$$= \frac{1}{4} \sec u + C = \frac{1}{4} \sec 4x + C$$

34 . Evaluate the integral.

$$\int e^{2x} \, dx$$

Try $u = 2x \Rightarrow \frac{du}{dx} = 2 \Rightarrow du = 2dx$

$\frac{1}{2} du = dx$

$$\int e^{2x} dx = \int e^{u} \cdot \frac{1}{2} du = \frac{1}{2} \int e^{u} du$$

$$= \frac{1}{2} e^{u} + C = \frac{1}{2} e^{2x} + C$$

35. Evaluate the integral.

$$\int x^2 e^{-2x^3}\,dx$$

Try the substitution $u = -2x^3$

Then $\frac{du}{dx} = -6x^2$

$du = -6x^2\,dx$

$-\frac{1}{6}\,du = x^2\,dx$

$$\int e^{-2x^3} x^2\,dx = \int e^u \cdot \left(-\frac{1}{6}\right) du$$

$$= -\frac{1}{6}\int e^u\,du = -\frac{1}{6} e^u + C$$

$$= -\frac{1}{6} e^{-2x^3} + C$$