

WRITE YOUR NAME:

MAC 2311 Homework 2
Due in class, Friday September 22nd
You can use more paper if necessary, but please STAPLE

Question 1. Guess the approximate value of

$$\lim_{x \rightarrow 3} \frac{2^x - 8}{x - 3}$$

by evaluating at some specific values of x . You may use a calculator or computer for this question.

With the help of a computer, we find

$$\frac{2^{3.001} - 8}{3.001 - 3} \approx 5.5471$$

$$\frac{2^{3.0001} - 8}{3.0001 - 3} \approx 5.5454$$

$$\frac{2^{2.999} - 8}{2.999 - 3} \approx 5.5433$$

$$\frac{2^{2.9999} - 8}{2.9999 - 3} \approx 5.5450$$

The limit
appears to be
around 5.545
(Exact value turns out
to be $8 \ln 2$ btw)

Question 2. Evaluate the limit.

$$\lim_{x \rightarrow 0} \frac{\sqrt{x^2+4}-2}{x^2}$$

$$\lim_{x \rightarrow 0} \frac{\sqrt{x^2+4}-2}{x^2} \cdot \frac{\sqrt{x^2+4}+2}{\sqrt{x^2+4}+2}$$

$$= \lim_{x \rightarrow 0} \frac{(\sqrt{x^2+4})^2 - 2^2}{x^2(\sqrt{x^2+4}+2)} = \lim_{x \rightarrow 0} \frac{x^2+4-4}{x^2(\sqrt{x^2+4}+2)}$$

$$= \lim_{x \rightarrow 0} \frac{x^2}{x^2(\sqrt{x^2+4}+2)} = \lim_{x \rightarrow 0} \frac{1}{\sqrt{x^2+4}+2}$$

$$= \frac{1}{\sqrt{0+4}+2} = \frac{1}{\sqrt{4}+2} = \frac{1}{2+2} = \frac{1}{4}$$

Question 3. Evaluate the limit.

$$\lim_{x \rightarrow \infty} (\sqrt{x^2 + 10x} - x)$$

$$\lim_{x \rightarrow \infty} \frac{(\sqrt{x^2 + 10x} - x) \cdot (\sqrt{x^2 + 10x} + x)}{(\sqrt{x^2 + 10x} + x)}$$

$$= \lim_{x \rightarrow \infty} \frac{(\sqrt{x^2 + 10x})^2 - x^2}{\sqrt{x^2 + 10x} + x} = \lim_{x \rightarrow \infty} \frac{x^2 + 10x - x^2}{\sqrt{x^2 + 10x} + x}$$

$$= \lim_{x \rightarrow \infty} \frac{10x}{\sqrt{x^2 + 10x} + x} \cdot \frac{\frac{1}{x}}{\frac{1}{x}} = \lim_{x \rightarrow \infty} \frac{\frac{10x}{x}}{\frac{\sqrt{x^2 + 10x}}{x} + \frac{x}{x}}$$

$$= \lim_{x \rightarrow \infty} \frac{10}{\frac{\sqrt{x^2 + 10x}}{\sqrt{x^2}} + 1} = \lim_{x \rightarrow \infty} \frac{10}{\sqrt{\frac{x^2 + 10x}{x^2}} + 1}$$

$$= \lim_{x \rightarrow \infty} \frac{10}{\sqrt{\frac{x^2}{x^2} + \frac{10x}{x^2}} + 1} = \lim_{x \rightarrow \infty} \frac{10}{\sqrt{1 + \frac{10}{x}} + 1}$$

$$= \frac{10}{\sqrt{1+0} + 1} = \frac{10}{\sqrt{1} + 1} = \frac{10}{1+1} = \frac{10}{2} = 5$$