WRITE YOUR NAME:

MAC 2311 Homework 3
Due in class, Friday February 17th
You can use more paper if necessary, but please STAPLE

Question 1. For each function, find the horizontal asymptotes, if there are any.

$$f(x) = \frac{2x-7}{x^2-4x}$$

$$g(x) = \frac{2x^2-6}{x^2+5x}$$

$$f(x) = \frac{2x-7}{x^2-4x} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}} = \frac{\frac{2x}{x^2} - \frac{7}{x^2}}{\frac{x^2}{x^2} - \frac{4x}{x^2}}$$

$$= \frac{\frac{2}{x} - \frac{7}{x^2}}{1 - \frac{4}{x}}$$

So $f(x)$ approaches $\frac{0-0}{1-0} = \frac{0}{1} = 0$ if $x \to \pm\infty$.

$y = 0$ is a horizontal asymptote.

$$g(x) = \frac{2x^2-6}{x^2+5x} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}} = \frac{\frac{2x^2}{x^2} - \frac{6}{x^2}}{\frac{x^2}{x^2} + \frac{5x}{x^2}}$$

$$= \frac{2 - \frac{6}{x^2}}{1 + \frac{5}{x}}$$

So $g(x)$ approaches $\frac{2-0}{1+0} = \frac{2}{1} = 2$ if $x \to \pm\infty$.

$y = 2$ is a horizontal asymptote.

Question 2. Find the derivative of the function, and simplify.

$$f(x) = \left(\frac{1}{x} + \frac{1}{x^2}\right)(3x^3 + 27)$$

$$f(x) = (x^{-1} + x^{-2})(3x^3 + 27)$$

METHOD 1: Use product rule.

$$f'(x) = (x^{-1} + x^{-2})'(3x^3+27) + (x^{-1}+x^{-2})(3x^3+27)'$$

$$= (-1x^{-2} - 2x^{-3})(3x^3+27) + (x^{-1}+x^{-2})(9x^2)$$

Can leave answer in that form but can also exercise your algebraic skills by expanding:

$$-3x - 27x^{-2} - 6 - 54x^{-3} + 9x + 9$$

$$= 6x + 3 - 27x^{-2} - 54x^{-3}$$

METHOD 2: Expand $f(x)$ before taking derivative

$$f(x) = (x^{-1}+x^{-2})(3x^3+27) = 3x^2 + 27x^{-1} + 3x + 27x^{-2}$$

Then $f'(x) = 3\cdot 2x + 27\cdot(-1)x^{-2} + 3 + 27\cdot(-2)x^{-3}$

$$= 6x - 27x^{-2} + 3 - 54x^{-3}$$

Can also be written $\dfrac{6x^4 - 27x + 3x^3 - 54}{x^3}$

Also can factor out a 3

Question 3. Find the derivative of the function, and simplify.

$$g(x) = \frac{x^2 - 1}{x^4 + 1}$$

$$g'(x) = \frac{(x^2-1)'(x^4+1) - (x^2-1)(x^4+1)'}{(x^4+1)^2}$$

$$= \frac{2x \cdot (x^4+1) - (x^2-1) \cdot 4x^3}{(x^4+1)^2}$$

$$= \frac{2x^5 + 2x - (4x^5 - 4x^3)}{(x^4+1)^2}$$

$$= \frac{2x^5 + 2x - 4x^5 + 4x^3}{(x^4+1)^2}$$

$$= \frac{-2x^5 + 4x^3 + 2x}{(x^4+1)^2} \quad \text{or} \quad \frac{2x \cdot (-x^4 + 2x^2 + 1)}{(x^4+1)^2}$$

$$\text{or} \quad \frac{-2x \cdot (x^4 - 2x^2 - 1)}{(x^4+1)^2}$$