WRITE YOUR NAME:

MAC 2311 Homework 4
Due in class, Friday February 24th
You can use more paper if necessary, but please STAPLE

Question 1. Find $f'(x)$.

$$f(x) = \frac{2x^2+5}{3x-4}$$

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$$

$u = 2x^2+5 \Rightarrow u' = 4x$
$v = 3x-4 \Rightarrow v' = 3$

$$f'(x) = \frac{(2x^2+5)'(3x-4)-(2x^2+5)(3x-4)'}{(3x-4)^2}$$

$$= \frac{4x\cdot(3x-4)-(2x^2+5)\cdot 3}{(3x-4)^2}$$

Technically the question doesn't say "simplify" but it's good practice

$$= \frac{12x^2-16x-(6x^2+15)}{(3x-4)^2} = \frac{12x^2-16x-6x^2-15}{(3x-4)^2}$$

$$= \frac{6x^2-16x-15}{(3x-4)^2}$$

$$(uv)' = u'v + uv'$$

Question 2. Find $f'(x)$.

$$f(x) = \sec x \tan x$$

$u = \sec x \Rightarrow u' = \sec x \tan x$

$v = \tan x \Rightarrow v' = \sec^2 x$

$$f'(x) = (\sec x)' \tan x + \sec x (\tan x)'$$

$$= \sec x \tan x \tan x + \sec x \sec^2 x$$

$$= \sec x (\tan^2 x + \sec^2 x)$$

Can be written other ways

e.g. $\tan^2 x = \sec^2 x - 1$

so $f'(x) = \sec x (\sec^2 x - 1 + \sec^2 x)$

$$= \sec x (2\sec^2 x - 1)$$

$$= 2\sec^3 x - \sec x$$

Question 3. Find $f'(x)$.

$$f(x) = \frac{\sec x}{1+\tan x} \qquad \left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$$

$$f'(x) = \frac{(\sec x)'(1+\tan x) - (\sec x)(1+\tan x)'}{(1+\tan x)^2}$$

$$= \frac{\sec x \tan x (1+\tan x) - \sec x \cdot \sec^2 x}{(1+\tan x)^2}$$

$$= \frac{\sec x \tan x + \sec x \tan^2 x - \sec x \sec^2 x}{(1+\tan x)^2}$$

$$= \frac{\sec x \cdot (\tan x + \tan^2 x - \sec^2 x)}{(1+\tan x)^2}$$

$$= \frac{\sec x \cdot (\tan x - 1)}{(1+\tan x)^2}$$

FACT:

$$\tan^2 x - \sec^2 x$$

$$= \frac{\sin^2 x}{\cos^2 x} - \frac{1}{\cos^2 x}$$

$$= \frac{\sin^2 x - 1}{\cos^2 x}$$

$$= \frac{-(1-\sin^2 x)}{\cos^2 x} = \frac{-\cos^2 x}{\cos^2 x}$$

$$= -1$$

Question 4. Find $f'(x)$.

$$f(x) = (3x^2 + 2x - 1)^6$$

$$f = (3x^2 + 2x - 1)^6$$

$$f = u^6 \quad \text{and} \quad u = 3x^2 + 2x - 1$$

$$\downarrow \qquad\qquad\qquad \downarrow$$

$$\frac{df}{du} = 6u^5 \qquad\qquad \frac{du}{dx} = 6x + 2$$

$$f'(x) = \frac{df}{dx} = \frac{df}{du} \cdot \frac{du}{dx} = 6u^5 \cdot (6x+2)$$

$$= \boxed{6(3x^2 + 2x - 1)^5 \cdot (6x+2)}$$

Can also be written $6(3x^2 + 2x - 1)^5 \cdot 2(3x+1)$

$$12(3x^2 + 2x - 1)^5 (3x+1)$$

ALTERNATIVE SOLUTION. Very short if you have enough practice with the chain rule.

$$f'(x) = 6(3x^2 + 2x - 1)^5 \cdot (3x^2 + 2x - 1)'$$

$$= 6(3x^2 + 2x - 1)^5 \cdot (6x + 2)$$

Question 5. Find $f'(x)$.

$$f(x) = \sin\left(\frac{1}{x^2}\right)$$

$$f(x) = \sin\left(x^{-2}\right)$$

$f = \sin u$ and $u = x^{-2}$

$$\frac{df}{du} = \cos u \qquad \frac{du}{dx} = -2x^{-3}$$

$$f'(x) = \frac{df}{dx} = \frac{df}{du}\cdot\frac{du}{dx} = \cos u \cdot \left(-2x^{-3}\right)$$

$$= \boxed{\cos\left(x^{-2}\right)\cdot\left(-2x^{-3}\right)}$$

which can also be written $\cos\left(\frac{1}{x^2}\right)\cdot\frac{-2}{x^3}$ or $\frac{-2}{x^3}\cos\left(\frac{1}{x^2}\right)$

ALTERNATIVE SOLUTION. Very quick with practice.

$$f(x) = \sin\left(x^{-2}\right)$$

$$f'(x) = \cos\left(x^{-2}\right)\cdot\left(x^{-2}\right)'$$

$$= \cos\left(x^{-2}\right)\cdot\left(-2x^{-3}\right)$$