

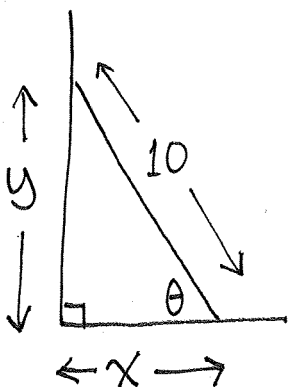
WRITE YOUR NAME:

MAC 2311 Homework 5

Due in class, Friday March 24th

You can use more paper if necessary, but please STAPLE

Question 1. A 10 foot ladder is leaning against a wall. The bottom of the ladder is being pushed toward the wall at a rate of 1 foot per second. If θ is the angle that the ladder makes with the ground, how fast is θ increasing at the moment when the bottom of the ladder is 6 feet from the wall?



x and θ are functions of time t

x is decreasing

θ is increasing

$$\text{Given: } \frac{dx}{dt} = -1$$

WANT: Rate $\frac{d\theta}{dt}$ at the moment when $x = 6$

From our knowledge of trigonometry,

$$\text{we know } \cos \theta = \frac{x}{10} \quad \left(\frac{\text{adjacent}}{\text{hypotenuse}} \right)$$

$$\Rightarrow \frac{d}{dt}(\cos \theta) = \frac{d}{dt} \left(\frac{x}{10} \right)$$

$$-\sin \theta \cdot \frac{d\theta}{dt} = \frac{1}{10} \cdot \frac{dx}{dt}$$

$$\underbrace{\hspace{2cm}}_{\text{WANT}} \quad \underbrace{\hspace{2cm}}_{=-1}$$

What is $\sin \theta$ when $x = 6$?

$$x = 6 \Rightarrow x^2 + y^2 = 10^2$$
$$6^2 + y^2 = 10^2$$

$$\Rightarrow 36 + y^2 = 100 \Rightarrow y^2 = 64$$
$$\Rightarrow y = 8. \text{ Then } \sin \theta = \frac{y}{10} = \frac{8}{10}$$

$$\Rightarrow -\frac{8}{10} \cdot \frac{d\theta}{dt} = \frac{1}{10} \cdot (-1)$$

$$-\frac{8}{10} \cdot \frac{d\theta}{dt} = -\frac{1}{10}$$

$$\frac{d\theta}{dt} = -\frac{1}{10} \cdot -\frac{10}{8}$$

$$= \frac{1}{8}$$