WRITE YOUR NAME:

MAC 2311 Quiz 4
Wednesday March 29th

Question 1. Evaluate the limit.

$$\lim_{x\to 0}(e^x + x)^{1/x}$$

Remember to do ONE STEP AT A TIME!

Let $L = (e^x + x)^{1/x}$. We want $\lim\limits_{x\to 0} L$

$$\ln L = \ln\left((e^x+x)^{1/x}\right) = \frac{1}{x}\ln(e^x+x)$$

$$\ln L = \frac{\ln(e^x+x)}{x}$$

If $x \to 0$, then bottom $\to 0$
Top $\to \ln(e^0+0) = \ln(1) = 0$
So this is of the form $\frac{0}{0}$

$$\lim_{x\to 0}\ln L = \lim_{x\to 0}\frac{\ln(e^x+x)}{x} \underset{\text{L'Hopital}}{=} \lim_{x\to 0}\frac{\frac{d}{dx}\ln(e^x+x)}{\frac{d}{dx}(x)}$$

CHAIN RULE

$$= \lim_{x\to 0}\frac{\frac{1}{e^x+x}\cdot(e^x+x)'}{1} = \lim_{x\to 0}\frac{1}{e^x+x}\cdot(e^x+1)$$

$$= \lim_{x\to 0}\frac{e^x+1}{e^x+x} = \frac{e^0+1}{e^0+0} = \frac{1+1}{1+0} = \frac{2}{1} = 2$$

So $\ln L$ approaches 2, so L approaches $\boxed{e^2}$