WRITE YOUR NAME:

MAC 2311 Section U08 Test 1
Friday February 10th
Total possible score: 20 points (2 points per page)

Question 2. Evaluate the limit.

$$\lim_{x \to 5} \frac{4x + 7}{x - 2}$$

$$\frac{4 \cdot 5 + 7}{5 - 2} = \frac{20 + 7}{3} = \frac{27}{3} = 9$$

Question 2. Evaluate the limit.

$$\lim_{x \to \infty} \frac{x^2 + 1}{x^3 + x}$$

$$\lim_{x \to \infty} \frac{x^2 + 1}{x(x^2 + 1)}$$

$$= \lim_{x \to \infty} \frac{1}{x} = 0$$

Question 3. Evaluate the limit.

$$\lim_{x \to -1} \frac{x^2 + 6x + 5}{x^2 - 3x - 4}$$

$$\lim_{x \to -1} \frac{(x+1)(x+5)}{(x+1)(x-4)}$$

$$= \lim_{x \to -1} \frac{x+5}{x-4}$$

$$= \frac{-1+5}{-1-4} = \frac{4}{-5} = -\frac{4}{5}$$

Question 4. Evaluate the one-sided limits. Specify whether each limit is $+\infty$ or $-\infty$.

$$\lim_{x\to -3^-} \frac{x}{x^2-9} \qquad \lim_{x\to -3^+} \frac{x}{x^2-9}$$

When $x \to -3^-$, x is slightly less than -3

$$x < -3$$

$$x+3 < 0$$

$$\frac{x}{x^2-9} = \frac{x}{(x+3)(x-3)} = \frac{\text{near } -3}{(\text{near } 0 \text{ and negative})(\text{near } -6)} \qquad \frac{\text{NEG}}{\text{NEG}\cdot\text{NEG}}$$

$$\lim_{x\to -3^-} \frac{x}{x^2-9} = -\infty$$

When $x \to -3^+$, x is slightly greater than -3

$$x > -3$$

$$x+3 > 0$$

$$\frac{x}{x^2-9} = \frac{x}{(x+3)(x-3)} = \frac{\text{near } -3}{(\text{near } 0 \text{ and positive})(\text{near } -6)} \qquad \frac{\text{NEG}}{\text{POS}\cdot\text{NEG}}$$

$$\lim_{x\to -3^+} \frac{x}{x^2-9} = +\infty$$

Question 5. Evaluate the limit.

$$\lim_{x\to 0} \frac{\sqrt{x+9}-3}{x}$$

$$= \lim_{x\to 0} \frac{\left(\sqrt{x+9}-3\right)\left(\sqrt{x+9}+3\right)}{x\left(\sqrt{x+9}+3\right)}$$

$$= \lim_{x\to 0} \frac{\left(\sqrt{x+9}\right)^2 - 3^2}{x\left(\sqrt{x+9}+3\right)} = \lim_{x\to 0} \frac{x+9-9}{x\left(\sqrt{x+9}+3\right)}$$

$$= \lim_{x\to 0} \frac{x}{x\left(\sqrt{x+9}+3\right)}$$

$$= \lim_{x\to 0} \frac{1}{\sqrt{x+9}+3} = \frac{1}{\sqrt{0+9}+3}$$

$$= \frac{1}{\sqrt{9}+3} = \frac{1}{3+3} = \frac{1}{6}$$

Question 6. Evaluate the limit.

$$\lim_{x \to -\infty} \frac{\sqrt{3x^4 + x}}{x^2 - 8}$$

$$= \lim_{x \to -\infty} \frac{\sqrt{3x^4+x}}{x^2-8} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}}$$

Note: $x^2 = \sqrt{x^4}$

$$= \lim_{x \to -\infty} \frac{\frac{\sqrt{3x^4+x}}{\sqrt{x^4}}}{\frac{x^2-8}{x^2}} = \lim_{x \to -\infty} \frac{\sqrt{\frac{3x^4+x}{x^4}}}{\frac{x^2-8}{x^2}}$$

$$= \lim_{x \to -\infty} \frac{\sqrt{3 + \frac{1}{x^3}}}{1 - \frac{8}{x^2}} = \frac{\sqrt{3+0}}{1-0}$$

$$= \frac{\sqrt{3}}{1} = \sqrt{3}$$

Question 7. Find all values of x (if any) at which $f(x)$ is not continuous, and determine whether each discontinuity is a removable discontinuity.

$$f(x) = \frac{5}{x} + \frac{x+2}{x^2-4}$$

$$f(x) = \frac{5}{x} + \frac{x+2}{(x+2)(x-2)}$$

Discontinuous at $x=0$, $x=-2$, $x=2$

The discontinuity at $x=-2$ is removable (because $\lim_{x \to -2} f(x)$ exists)

The other discontinuities are not removable.

Question 8. Find the discontinuities, if any.

$$f(x) = \frac{3 - 5\cos x}{\sin x}$$

Discontinuous when $\sin x = 0$

which happens when $x = 0, \pm\pi, \pm 2\pi, \pm 3\pi, \ldots$

Question 9. Find the limit.

$$\lim_{x\to\infty} \cos\left(\frac{\pi x^2+1}{x^2+2017}\right)$$

$$\lim_{x\to\infty} \cos\left(\frac{\pi x^2+1}{x^2+2017}\cdot\frac{\frac{1}{x^2}}{\frac{1}{x^2}}\right)$$

$$= \lim_{x\to\infty} \cos\left(\frac{\pi+\frac{1}{x^2}}{1+\frac{2017}{x^2}}\right)$$

$$= \cos\left(\frac{\pi+0}{1+0}\right).$$

$$= \cos\pi$$

$$= -1$$

Question 10a. Write down the **definition** of the derivative.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Question 10b. Use the **definition** of the derivative to find the derivative of $f(x) = \sqrt{x}$.

$$f'(x) = \lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} \cdot \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}}$$

$$= \lim_{h \to 0} \frac{(\sqrt{x+h})^2 - (\sqrt{x})^2}{h(\sqrt{x+h} + \sqrt{x})} = \lim_{h \to 0} \frac{x+h-x}{h(\sqrt{x+h} + \sqrt{x})}$$

$$= \lim_{h \to 0} \frac{h}{h(\sqrt{x+h} + \sqrt{x})} = \lim_{h \to 0} \frac{1}{\sqrt{x+h} + \sqrt{x}}$$

$$= \frac{1}{\sqrt{x+0} + \sqrt{x}} = \frac{1}{\sqrt{x} + \sqrt{x}} = \frac{1}{2\sqrt{x}}$$