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MAC 2311 Section U10 Test 1
Wednesday October 4th
Total possible score: 20 points (2 points per page)

Question 1. Evaluate the limit.

$$\lim_{x\to 5} \frac{4x+7}{x-2}$$

Nothing weird happens near $x=5$

$$\lim_{x\to 5} \frac{4x+7}{x-2} = \frac{4\cdot 5+7}{5-2} = \frac{20+7}{3} = \frac{27}{3} = 9$$

Question 2. Evaluate the limit.

$$\lim_{x\to\infty} \frac{x^2+1}{x^3+x}$$

Quickest way: $\dfrac{\deg 2}{\deg 3}$ and $x \to \infty$

means limit is 0

Also note $\dfrac{x^2+1}{x^3+x} = \dfrac{x^2+1}{x(x^2+1)} = \dfrac{1}{x}$

So $\displaystyle\lim_{x\to\infty} \frac{x^2+1}{x^3+x} = \lim_{x\to\infty} \frac{1}{x} = 0$

Also could say $\dfrac{x^2+1}{x^3+x} \cdot \dfrac{\frac{1}{x^3}}{\frac{1}{x^3}} = \dfrac{\frac{1}{x}+\frac{1}{x^3}}{1+\frac{1}{x^2}}$

Question 3. Evaluate the limit.

$$\lim_{x\to -1} \frac{x^2+6x+5}{x^2-3x-4}$$

Can factor by trial and error

$$x^2+6x+5 = (x+1)(x+5)$$

$$x^2-3x-4 = (x+1)(x-4)$$

$$\lim_{x\to -1} \frac{x^2+6x+5}{x^2-3x-4} = \lim_{x\to -1} \frac{(x+1)(x+5)}{(x+1)(x-4)}$$

$$= \lim_{x\to -1} \frac{x+5}{x-4} = \frac{-1+5}{-1-4} = \frac{4}{-5} \text{ or } -\frac{4}{5}$$

Question 4. Evaluate the one-sided limits. Specify whether each limit is $+\infty$ or $-\infty$.

$$\lim_{x\to -3^-} \frac{x}{x^2-9} \qquad \lim_{x\to -3^+} \frac{x}{x^2-9}$$

$x \to -3^- \Rightarrow x$ slightly less than -3 (e.g. $x = -3.001$)

$\Rightarrow x^2 - 9$ is near 0 and positive

Numerator x is near -3 and negative

$$\lim_{x\to -3^-} \frac{x}{x^2-9} = -\infty \qquad \left(\frac{\text{negative near } -3}{\text{positive near } 0}\right)$$

$x \to -3^+ \Rightarrow x$ slightly more than -3 (e.g. $x = -2.999$)

$\Rightarrow x^2 - 9$ is near 0 and negative

Numerator x is near -3 and negative

$$\lim_{x\to -3^+} \frac{x}{x^2-9} = +\infty \qquad \left(\frac{\text{negative near } -3}{\text{negative near } 0}\right)$$

Note: $(-3.001)^2 = (3.001)^2 = 9.00$something (slightly more than 9)

$(-2.999)^2 = (2.999)^2 =$ slightly less than 9

Question 5. Evaluate the limit.

$$\lim_{x\to 0} \frac{\sqrt{x+9}-3}{x}$$

$$\lim_{x\to 0} \frac{\sqrt{x+9}-3}{x} \cdot \frac{\sqrt{x+9}+3}{\sqrt{x+9}+3}$$

$$= \lim_{x\to 0} \frac{(\sqrt{x+9})^2-3^2}{x(\sqrt{x+9}+3)} = \lim_{x\to 0} \frac{x+9-9}{x(\sqrt{x+9}+3)}$$

$$= \lim_{x\to 0} \frac{x}{x(\sqrt{x+9}+3)} = \lim_{x\to 0} \frac{1}{\sqrt{x+9}+3}$$

$$= \frac{1}{\sqrt{0+9}+3} = \frac{1}{\sqrt{9}+3} = \frac{1}{3+3} = \frac{1}{6}$$

Question 6. Evaluate the limit.

$$\lim_{x \to -\infty} \frac{\sqrt{3x^4 + x}}{x^2 - 8}$$

Note: $\sqrt{3x^4+x}$ is slightly more than $\sqrt{3x^4} = \sqrt{3} \cdot x^2$ which is about the same size as a multiple of x^2.

$$\lim_{x \to -\infty} \frac{\sqrt{3x^4+x}}{x^2-8} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}}$$

$$= \lim_{x \to -\infty} \frac{\frac{\sqrt{3x^4+x}}{x^2}}{\frac{x^2-8}{x^2}} = \lim_{x \to -\infty} \frac{\frac{\sqrt{3x^4+x}}{\sqrt{x^4}}}{\frac{x^2-8}{x^2}}$$

$$= \lim_{x \to -\infty} \frac{\sqrt{\frac{3x^4+x}{x^4}}}{\frac{x^2-8}{x^2}} = \lim_{x \to -\infty} \frac{\sqrt{3+\frac{1}{x^3}}}{1-\frac{8}{x^2}}$$

$$= \frac{\sqrt{3+0}}{1-0} = \sqrt{3}$$

Question 7. Find all values of x (if any) at which $f(x)$ is not continuous, and determine whether each discontinuity is a removable discontinuity.

$$f(x) = \frac{5}{x} + \frac{x+2}{x^2-4}$$

f is discontinuous at points where we divide by 0

so f is discontinuous at $x=0$, $x=2$, $x=-2$

At $x=0$, $\frac{5}{x}$ is $\frac{\text{nonzero}}{\text{zero}}$ $\rightarrow$ infinite discontinuity
not removable

At $x=2$, $\frac{x+2}{x^2-4}$ is $\frac{\text{nonzero}}{\text{zero}}$ $\rightarrow$ infinite discontinuity
not removable

At $x=-2$ ...

$$\lim_{x\to-2} \frac{5}{x} = \frac{5}{-2} = \text{just a number}$$

$$\lim_{x\to-2} \frac{x+2}{x^2-4} = \lim_{x\to-2} \frac{x+2}{(x+2)(x-2)} = \lim_{x\to-2} \frac{1}{x-2}$$

$$= \frac{1}{-2-2} = \frac{1}{-4} \quad \text{just a number}$$

So $\lim\limits_{x\to-2} f(x)$ exists, so the discontinuity at $x=-2$
is removable

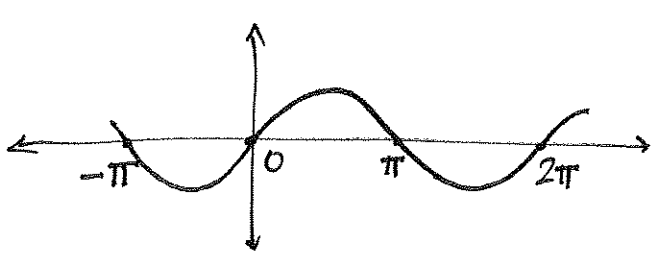
Question 8. Find the discontinuities, if any.

$$f(x) = \frac{3 - 5\cos x}{\sin x}$$

Discontinuous when $\sin x = 0$

so when $x = 0, \pm\pi, \pm 2\pi, \pm 3\pi, \ldots$

$y = \sin x$:



Question 9. Find the limit.

$$\lim_{x\to\infty} \cos\left(\frac{\pi x^2+1}{x^2+2017}\right)$$

If $x \to \infty$, then $\dfrac{\pi x^2 + 1}{x^2 + 2017} \to \pi$

So then $\cos\left(\dfrac{\pi x^2+1}{x^2+2017}\right) \to \cos\pi = -1$

Question 10a. Write down the **definition** of the derivative.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

Question 10b. Use the **definition** of the derivative to find the derivative of $f(x) = \sqrt{x}$.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\sqrt{x+h}-\sqrt{x}}{h}$$

$$= \lim_{h\to 0} \frac{\sqrt{x+h}-\sqrt{x}}{h} \cdot \frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}$$

$$= \lim_{h\to 0} \frac{\left(\sqrt{x+h}\right)^2-\left(\sqrt{x}\right)^2}{h\left(\sqrt{x+h}+\sqrt{x}\right)} = \lim_{h\to 0} \frac{x+h-x}{h\left(\sqrt{x+h}+\sqrt{x}\right)}$$

$$= \lim_{h\to 0} \frac{h}{h\left(\sqrt{x+h}+\sqrt{x}\right)} = \lim_{h\to 0} \frac{1}{\sqrt{x+h}+\sqrt{x}} = \frac{1}{\sqrt{x+0}+\sqrt{x}}$$

$$= \frac{1}{\sqrt{x}+\sqrt{x}} = \frac{1}{2\sqrt{x}}$$