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MAC 2311 Section U20 Test 1
Friday October 6th
Total possible score: 20 points (2 points per page)

Question 1. Evaluate the limit.

$$\lim_{x \to 4} \frac{5x + 4}{x - 1}$$

Nothing weird happens near $x = 4$

$$\lim_{x \to 4} \frac{5x+4}{x-1} = \frac{5 \cdot 4 + 4}{4-1} = \frac{20+4}{3} = \frac{24}{3} = 8$$

Question 2. Evaluate the limit.

$$\lim_{x\to\infty} \frac{x^2+1}{x^3+x}$$

Quickest way: $\dfrac{\text{deg } 2}{\text{deg } 3}$ and $x \to \infty$

means limit is 0

Also note $\dfrac{x^2+1}{x^3+x} = \dfrac{x^2+1}{x(x^2+1)} = \dfrac{1}{x}$

So $\displaystyle\lim_{x\to\infty} \frac{x^2+1}{x^3+x} = \lim_{x\to\infty} \frac{1}{x} = 0$

Also could say $\dfrac{x^2+1}{x^3+x} \cdot \dfrac{\frac{1}{x^3}}{\frac{1}{x^3}} = \dfrac{\frac{1}{x} + \frac{1}{x^3}}{1 + \frac{1}{x^2}}$

Question 3. Evaluate the limit.

$$\lim_{x\to -2} \frac{x^2+5x+6}{x^2-2x-8}$$

Can factor by trial and error

$$x^2+5x+6 = (x+2)(x+3)$$
$$x^2-2x-8 = (x+2)(x-4)$$

$$\lim_{x\to -2} \frac{x^2+5x+6}{x^2-2x-8} = \lim_{x\to -2} \frac{(x+2)(x+3)}{(x+2)(x-4)}$$

$$= \lim_{x\to -2} \frac{x+3}{x-4} = \frac{-2+3}{-2-4} = \frac{1}{-6} \text{ or } -\frac{1}{6}$$

Question 4. Evaluate the one-sided limits. Specify whether each limit is $+\infty$ or $-\infty$.

$$\lim_{x\to -5^-} \frac{x}{x^2-25} \qquad \lim_{x\to -5^+} \frac{x}{x^2-25}$$

$x \to -5^- \Rightarrow x$ slightly less than -5 (e.g. $x = -5.001$)

$\Rightarrow x^2 - 25$ is near 0 and positive

Numerator x is near -5 and negative

$$\lim_{x\to -5^-} \frac{x}{x^2-25} = -\infty \quad \left(\frac{\text{negative near } -5}{\text{positive near } 0} \right)$$

$x \to -5^+ \Rightarrow x$ slightly more than -5 (e.g. $x = -4.999$)

$\Rightarrow x^2 - 25$ is near 0 and negative

Numerator x is near -5 and negative

$$\lim_{x\to -5^+} \frac{x}{x^2-25} = +\infty \quad \left(\frac{\text{negative near } -5}{\text{negative near } 0} \right)$$

Note: $(-5.001)^2 = (5.001)^2 = 25.\text{something}$ (slightly more than 25)

$(-4.999)^2 = (4.999)^2 =$ slightly less than 25

Question 5. Evaluate the limit.

$$\lim_{x\to 0} \frac{\sqrt{x+16}-4}{x}$$

$$\lim_{x\to 0} \frac{\sqrt{x+16}-4}{x} \cdot \frac{\sqrt{x+16}+4}{\sqrt{x+16}+4}$$

$$= \lim_{x\to 0} \frac{\left(\sqrt{x+16}\right)^2 - 4^2}{x\left(\sqrt{x+16}+4\right)} = \lim_{x\to 0} \frac{x+16-16}{x\left(\sqrt{x+16}+4\right)}$$

$$= \lim_{x\to 0} \frac{x}{x\left(\sqrt{x+16}+4\right)} = \lim_{x\to 0} \frac{1}{\sqrt{x+16}+4}$$

$$= \frac{1}{\sqrt{0+16}+4} = \frac{1}{\sqrt{16}+4} = \frac{1}{4+4} = \frac{1}{8}$$

Question 6. Evaluate the limit.

$$\lim_{x \to -\infty} \frac{\sqrt{5x^4 + x}}{x^2 - 7}$$

Note: $\sqrt{5x^4+x}$ is slightly more than $\sqrt{5x^4} = \sqrt{5} \cdot x^2$ which is about the same size as a multiple of x^2.

$$\lim_{x \to -\infty} \frac{\sqrt{5x^4+x}}{x^2-7} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}}$$

$$= \lim_{x \to -\infty} \frac{\frac{\sqrt{5x^4+x}}{x^2}}{\frac{x^2-7}{x^2}} = \lim_{x \to -\infty} \frac{\frac{\sqrt{5x^4+x}}{\sqrt{x^4}}}{\frac{x^2-7}{x^2}}$$

$$= \lim_{x \to -\infty} \frac{\sqrt{\frac{5x^4+x}{x^4}}}{\frac{x^2-7}{x^2}} = \lim_{x \to -\infty} \frac{\sqrt{5+\frac{1}{x^3}}}{1-\frac{7}{x^2}}$$

$$= \frac{\sqrt{5+0}}{1-0} = \sqrt{5}$$

Question 7. Find all values of x (if any) at which $f(x)$ is not continuous, and determine whether each discontinuity is a removable discontinuity.

$$f(x) = \frac{7}{x} + \frac{x+3}{x^2-9}$$

f is discontinuous at points where we divide by 0

so f is discontinuous at $x=0$, $x=3$, $x=-3$

At $x=0$, $\frac{7}{x}$ is $\frac{\text{nonzero}}{\text{zero}}$ $\rightarrow$ infinite discontinuity
<u>not</u> removable

At $x=3$, $\frac{x+3}{x^2-9}$ is $\frac{\text{nonzero}}{\text{zero}}$ $\rightarrow$ infinite discontinuity
<u>not</u> removable

At $x=-3$ ...

$$\lim_{x\to-3} \frac{7}{x} = \frac{7}{-3} = \text{just a number}$$

$$\lim_{x\to-3} \frac{x+3}{x^2-9} = \lim_{x\to-3} \frac{x+3}{(x+3)(x-3)} = \lim_{x\to-3} \frac{1}{x-3}$$

$$= \frac{1}{-3-3} = \frac{1}{-6} \quad \text{just a number}$$

So $\lim_{x\to-3} f(x)$ exists, so the discontinuity at $x=-3$
is removable

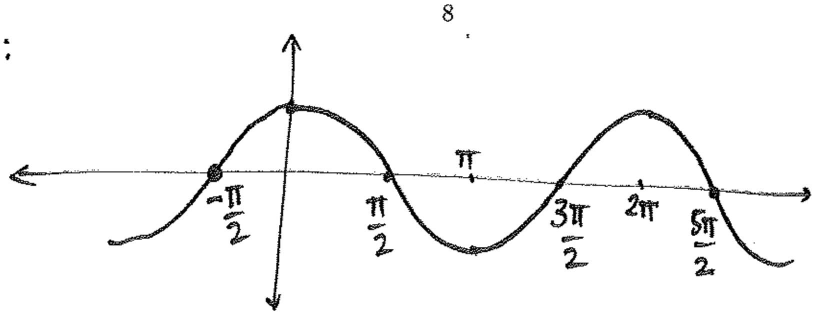
Question 8. Find the discontinuities, if any.

$$f(x) = \frac{3 - 5\sin x}{\cos x}$$

Discontinuous when $\cos x = 0$

So when $x = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \ldots$

$y = \cos x$:



Question 9. Find the limit.

$$\lim_{x\to\infty} \sin\left(\frac{\pi x^2+1}{2x^2+2017}\right)$$

If $x \to \infty$, then $\dfrac{\pi x^2+1}{2x^2+2017} \to \dfrac{\pi}{2}$

so then $\sin\left(\dfrac{\pi x^2+1}{2x^2+2017}\right) \to \sin\dfrac{\pi}{2} = 1$

Question 10a. Write down the **definition** of the derivative.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Question 10b. Use the **definition** of the derivative to find the derivative of $f(x) = \frac{1}{x}$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h} \cdot \frac{x(x+h)}{x(x+h)}$$

$$= \lim_{h \to 0} \frac{\frac{x(x+h)}{x+h} - \frac{x(x+h)}{x}}{hx(x+h)} = \lim_{h \to 0} \frac{x - (x+h)}{hx(x+h)}$$

$$= \lim_{h \to 0} \frac{x - x - h}{hx(x+h)} = \lim_{h \to 0} \frac{-h}{hx(x+h)} = \lim_{h \to 0} \frac{-1}{x(x+h)}$$

$$= \frac{-1}{x(x+0)} = \frac{-1}{x^2}$$