# MAC2311 Section U20
# Suggested problems for Test 1
# (Test 1 is Friday October 6th, in class)

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**1.** Evaluate the limit.

$$\lim_{x\to 0} \frac{6x-9}{x^3-12x+3}$$

Nothing prevents us from plugging in $x=0$

$$\frac{0-9}{0-0+3} = \frac{-9}{3} = -3$$

**2.** Evaluate the limit.

$$\lim_{x\to 1} \frac{x^4-1}{x-1}$$

Difference of squares

$$\lim_{x\to 1} \frac{(x^2+1)(x^2-1)}{x-1} = \lim_{x\to 1} \frac{(x^2+1)(x+1)(x-1)}{x-1} = \lim_{x\to 1} (x^2+1)(x+1)$$

$$= (1+1)(1+1) = 4$$

**3.** Evaluate the limit.

$$\lim_{x\to 2} \frac{x^2-4x+4}{x^2+x-6}$$

Can factor by trial and error

$$\lim_{x\to 2} \frac{(x-2)(x-2)}{(x+3)(x-2)} = \lim_{x\to 2} \frac{x-2}{x+3} = \frac{2-2}{2+3} = \frac{0}{5} = 0$$

4. Evaluate the limit.

$$\lim_{x \to 3^+} \frac{x}{x-3} \qquad \frac{\text{NONZERO}}{\text{ZERO}}$$

$x \to 3^+$
$x > 3$
$x - 3 > 0$

Top approaches 3, which is positive
Bottom approaches 0 and is positive

Answer: $+\infty$ or "does not exist"

5. Evaluate the limit.

$$\lim_{x \to 3^-} \frac{x}{x-3} \qquad \frac{\text{NONZERO}}{\text{ZERO}}$$

$x \to 3^-$
$x < 3$
$x - 3 < 0$

Top approaches 3, which is positive
Bottom approaches 0 and is negative

Answer: $-\infty$ or "does not exist"

6. Evaluate the limit.

$$\lim_{x \to 2^+} \frac{x}{x^2-4} = \lim_{x \to 2^+} \frac{x}{(x-2)(x+2)} \qquad \frac{\text{NONZERO}}{\text{ZERO}}$$

When $x \to 2^+$, $x$ is positive and approaches 2
$x-2$ is positive and approaches 0
$x+2$ is positive and approaches 4

Answer: $+\infty$ or "does not exist"

7. Evaluate the limit.

$$\lim_{x \to 2^-} \frac{x}{x^2-4} = \lim_{x \to 2^-} \frac{x}{(x-2)(x+2)} \qquad \frac{\text{NONZERO}}{\text{ZERO}}$$

When $x \to 2^-$, $x$ is positive and approaches 2
$x-2$ is negative and approaches 0
$x+2$ is positive and approaches 4



So whole thing is $\frac{\text{POS}}{\text{NEG} \cdot \text{POS}}$ = NEG and also $\frac{\text{NONZERO}}{\text{ZERO}}$

Answer: $-\infty$ or "does not exist"

$\frac{\text{zero}}{\text{zero}} \to$ don't know yet...

8. Evaluate the limit.

$$\lim_{x\to 9} \frac{x-9}{\sqrt{x}-3}$$ Try multiplying by "conjugate"

$$\lim_{x\to 9} \frac{(x-9)(\sqrt{x}+3)}{(\sqrt{x}-3)(\sqrt{x}+3)} = \lim_{x\to 9} \frac{(x-9)(\sqrt{x}+3)}{(\sqrt{x})^2-3^2}$$

$$= \lim_{x\to 9} \frac{(x-9)(\sqrt{x}+3)}{x-9} = \lim_{x\to 9} (\sqrt{x}+3) = \sqrt{9}+3$$

$$= 3+3 = 6$$

9. Evaluate the limit.

Conjugate? $$\lim_{x\to 0} \frac{\sqrt{x+4}-2}{x}$$ $\frac{\text{zero}}{\text{zero}}$ Don't know yet...

$$\lim_{x\to 0} \frac{(\sqrt{x+4}-2)(\sqrt{x+4}+2)}{x(\sqrt{x+4}+2)} = \lim_{x\to 0} \frac{(x+4)-4}{x(\sqrt{x+4}+2)}$$

$$= \lim_{x\to 0} \frac{x}{x(\sqrt{x+4}+2)} = \lim_{x\to 0} \frac{1}{\sqrt{x+4}+2} = \frac{1}{\sqrt{0+4}+2}$$

$$= \frac{1}{\sqrt{4}+2} = \frac{1}{2+2} = \frac{1}{4}$$

10. Evaluate the limit.

$$\lim_{x\to\infty} \frac{3x+1}{2x-5}$$

$$\lim_{x\to\infty} \frac{(3x+1)\cdot\frac{1}{x}}{(2x-5)\cdot\frac{1}{x}} = \lim_{x\to\infty} \frac{3+\frac{1}{x}}{2-\frac{5}{x}}$$

$$= \frac{3+0}{2-0} = \frac{3}{2}$$

**11.** Evaluate the limit.

$$\lim_{y\to-\infty} \frac{3}{y+4} \qquad \frac{\text{Finite}}{\text{Infinite}}$$

$$0$$

**12.** Evaluate the limit.

$$\lim_{x\to-\infty} \frac{x-2}{x^2+2x+1}$$

$$\lim_{x\to-\infty} \frac{(x-2)\cdot\frac{1}{x^2}}{(x^2+2x+1)\cdot\frac{1}{x^2}} = \lim_{x\to-\infty} \frac{\frac{1}{x}-\frac{2}{x^2}}{1+\frac{2}{x}+\frac{1}{x^2}}$$

$$= \frac{0-0}{1+0+0} = \frac{0}{1} = 0$$

**13.** Evaluate the limit.

$$\lim_{x\to\infty} \frac{5x^2+7}{3x^2-x}$$

$$\lim_{x\to\infty} \frac{(5x^2+7)\cdot\frac{1}{x^2}}{(3x^2-x)\cdot\frac{1}{x^2}} = \lim_{x\to\infty} \frac{5+\frac{7}{x^2}}{3-\frac{1}{x}} = \frac{5+0}{3-0} = \frac{5}{3}$$

**14.** Evaluate the limit.

$$\lim_{t\to-\infty} \frac{5-2t^3}{t^2+1}$$

$$\lim_{t\to-\infty} \frac{(5-2t^3)\cdot\frac{1}{t^3}}{(t^2+1)\cdot\frac{1}{t^3}} = \lim_{t\to-\infty} \frac{\frac{5}{t^3}-2}{\frac{1}{t}+\frac{1}{t^3}}$$



Approaches $\frac{0-2}{0+0} = \frac{-2}{0}$ $\quad \frac{\text{Nonzero}}{\text{Zero}}$ Limit does not exist.

Also, if $t\to-\infty$, then $\frac{5-2t^3}{t^2+1} \approx \frac{-2t^3}{t^2} = -2t \to +\infty$

15. Evaluate the limit.

$$\lim_{x\to-\infty} \frac{x+4x^3}{1-x^2+7x^3}$$

$$\lim_{x\to-\infty} \frac{(x+4x^3)\cdot\frac{1}{x^3}}{(1-x^2+7x^3)\cdot\frac{1}{x^3}} = \lim_{x\to-\infty} \frac{\frac{1}{x^2}+4}{\frac{1}{x^3}-\frac{1}{x}+7}$$

$$= \frac{0+4}{0-0+7} = \frac{4}{7}$$

16. Evaluate the limit.

$$\lim_{x\to\infty} \sqrt{\frac{2-3x+4x^2}{1+9x^2}}$$

$$\lim_{x\to\infty} \sqrt{\frac{(2-3x+4x^2)\cdot\frac{1}{x^2}}{(1+9x^2)\cdot\frac{1}{x^2}}} = \lim_{x\to\infty} \sqrt{\frac{\frac{2}{x^2}-\frac{3}{x}+4}{\frac{1}{x^2}+9}}$$

$$= \sqrt{\frac{0-0+4}{0+9}} = \sqrt{\frac{4}{9}} = \frac{2}{3}$$

17. Evaluate the limit.

$$\lim_{x\to\infty} \frac{\sqrt{5x^2-2}}{x+3}$$

Idea: Top "really" grows like $\sqrt{x^2} = x$

$$\lim_{x\to\infty} \frac{\sqrt{5x^2-2}\cdot\frac{1}{x}}{(x+3)\cdot\frac{1}{x}} = \lim_{x\to\infty} \frac{\sqrt{5x^2-2}\cdot\sqrt{\frac{1}{x^2}}}{(x+3)\cdot\frac{1}{x}}$$

$$= \lim_{x\to\infty} \frac{\sqrt{(5x^2-2)\cdot\frac{1}{x^2}}}{(x+3)\cdot\frac{1}{x}} = \lim_{x\to\infty} \frac{\sqrt{5-\frac{2}{x^2}}}{1+\frac{3}{x}}$$

$$= \frac{\sqrt{5-0}}{1+0} = \frac{\sqrt{5}}{1} = \sqrt{5}$$

**18.** Find all values of $x$, if any, at which $f$ is not continuous.

$$f(x) = (x-8)^{1/3}$$

Continuous for all $x$.

(Composition of $x-8$, which is continuous everywhere, and cube root function, which is continuous everywhere.)

**19.** Find all values of $x$, if any, at which $f$ is not continuous.

$$f(x) = \frac{x+2}{x^2-4}$$

Discontinuous when $x^2-4=0$ $\quad x^2=4 \quad x=\pm 2$

(Alternatively: $x^2-4=0 \Rightarrow (x-2)(x+2)=0 \Rightarrow x = 2$ or $-2$)

**20.** Find all values of $x$, if any, at which $f$ is not continuous.

$$f(x) = \frac{x}{2x^2+x}$$

Discontinuous when $2x^2+x=0$

$x(2x+1)=0$

$x=0$ or $2x+1=0 \Rightarrow 2x=-1 \Rightarrow x = \frac{-1}{2}$

**21.** Find all values of $x$, if any, at which $f$ is not continuous.

$$f(x) = \frac{3}{x} + \frac{x-1}{x^2-1}$$

Discontinuous when $x=0$ or $x^2-1=0$

$x^2=1$

$x=1$ or $-1$



Discontinuous if $x=0$ or $1$ or $-1$

22. Find all values of $x$ (if any) at which $f$ is not continuous, and determine whether each discontinuity is a removable discontinuity.

$$f(x) = \frac{x^2+3x}{x+3}$$ Discontinuous at $x = -3$.

$$\lim_{x\to -3} \frac{x^2+3x}{x+3} = \lim_{x\to -3} \frac{x(x+3)}{x+3} = \lim_{x\to -3} x = -3$$

Limit exists
Removable.

23. Find the discontinuities, if any.

$$f(x) = \sin(x^2-2)$$

No discontinuities. Composition of polynomial $x^2-2$, which is continuous everywhere, and sine function, which is continuous everywhere.

24. Find the discontinuities, if any.

$$f(x) = \cos\left(\frac{x}{x-\pi}\right)$$

Cosine function is continuous everywhere, but $\frac{x}{x-\pi}$ is discontinuous when $x = \pi$.

25. Find the discontinuities, if any.

$$f(x) = |\cot x|$$ Recall $\cot x = \frac{\cos x}{\sin x}$

Discontinuous when $\sin x = 0$, i.e. when $x = 0, \pm\pi, \pm 2\pi, \pm 3\pi, \ldots$

26. Find the discontinuities, if any.

$$f(x) = \frac{1}{1+\sin^2 x}$$

No discontinuities. Discontinuous if $1 + \sin^2 x = 0$

i.e. if $\sin^2 x = -1$

$\sin x = \sqrt{-1}$ Undefined if using real numbers

Never happens

**27.** Find the limit.

$$\lim_{x\to\infty} \cos\left(\frac{1}{x}\right)$$

Since $\lim\limits_{x\to\infty} \frac{1}{x} = 0$, we conclude $\lim\limits_{x\to\infty} \cos\left(\frac{1}{x}\right)$

$= \cos 0 = 1$

**28.** Find the limit.

$$\lim_{x\to\infty} \sin\left(\frac{\pi x}{2-3x}\right)$$

$$\lim_{x\to\infty} \sin\left(\frac{\pi x \cdot \frac{1}{x}}{(2-3x)\cdot\frac{1}{x}}\right) = \lim_{x\to\infty} \sin\left(\frac{\pi}{\frac{2}{x}-3}\right)$$

$$= \sin\left(\frac{\pi}{0-3}\right) = \sin\left(-\frac{\pi}{3}\right) = \frac{-\sqrt{3}}{2}$$

**29.** Find the limit.

$$\lim_{x\to\infty} \sin^{-1}\left(\frac{x}{1-2x}\right)$$

$$\lim_{x\to\infty} \sin^{-1}\left(\frac{x\cdot\frac{1}{x}}{(1-2x)\cdot\frac{1}{x}}\right) = \lim_{x\to\infty} \sin^{-1}\left(\frac{1}{\frac{1}{x}-2}\right)$$

$$= \sin^{-1}\left(\frac{1}{0-2}\right) = \sin^{-1}\left(-\frac{1}{2}\right)$$

$$= -\pi/6$$

**30.** Find the limit.

$$\lim_{x\to\infty} \ln\left(\frac{x+1}{x}\right)$$

$$\lim_{x\to\infty} \ln\left(\frac{(x+1)\cdot\frac{1}{x}}{x\cdot\frac{1}{x}}\right) = \lim_{x\to\infty} \ln\left(\frac{1+\frac{1}{x}}{1}\right)$$

$$= \ln\left(\frac{1+0}{1}\right) = \ln\left(\frac{1}{1}\right) = \ln 1 = 0$$

31. Write down the definition of the derivative.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

32. Use the definition of the derivative to find the derivative of $f(x) = x^2$.

$$f'(x) = \lim_{h \to 0} \frac{(x+h)^2 - x^2}{h} = \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - x^2}{h}$$

$$= \lim_{h \to 0} \frac{2xh + h^2}{h} = \lim_{h \to 0} \frac{(2x+h)h}{h} = \lim_{h \to 0} (2x+h)$$

$$= 2x + 0 = 2x$$

33. Use the definition of the derivative to find the derivative of $f(x) = \frac{1}{x}$.

$$f'(x) = \lim_{h \to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h} \cdot \frac{(x+h)x}{(x+h)x}$$

$$= \lim_{h \to 0} \frac{x - (x+h)}{h(x+h)x} = \lim_{h \to 0} \frac{-h}{h(x+h)x}$$

$$= \lim_{h \to 0} \frac{-1}{(x+h)x} = \frac{-1}{(x+0)x} = \frac{-1}{x^2}$$