MAC2312
Suggested problems on Chapter 5 material (integration)

Idris Mercer

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1. Evaluate the sum.

$$\sum_{k=1}^{3} k^3$$

2. Evaluate the sum.

$$\sum_{j=2}^{6} (3j-1)$$

3. Evaluate the sum.

$$\sum_{i=-4}^{1} (i^2 - i)$$

4. Evaluate the sum.

$$\sum_{n=0}^{5} 1$$

5. Evaluate the sum.

$$\sum_{k=0}^{4} (-2)^k$$

6. Evaluate the sum.

$$\sum_{n=1}^{6} \sin n\pi$$

7. Write the expression in sigma notation but do not evaluate.

$$1 + 2 + 3 + \cdots + 10$$

8. Write the expression in sigma notation but do not evaluate.

$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5}$$

9. Use formulas given in the textbook (Theorem 5.4.2) to evaluate the sum.

$$\sum_{k=1}^{100} k$$

10. Use formulas given in the textbook (Theorem 5.4.2) to evaluate the sum.

$$\sum_{k=1}^{30} k(k-2)(k+2)$$

11. Express the sum in closed form.

$$\sum_{k=1}^{n} \frac{3k}{n}$$

12. Let f be the function $f(x) = 3x+1$ on the interval $[2, 6]$. Divide the interval into $n = 4$ subintervals of equal length and then compute

$$\sum_{k=1}^{4} f(x_k^*)\Delta x$$

if
(a) x_k^* is the left endpoint of each subinterval,
(b) x_k^* is the midpoint of each subinterval,
(c) x_k^* is the right endpoint of each subinterval.
Illustrate each part with a graph of f that includes the rectangles whose areas are represented in the sum.

13. Consider the sum

$$\sum_{k=1}^{4} \big((k+1)^3 - k^3\big).$$

Notice that this is

$$(2^3 - 1^3) + (3^3 - 2^3) + (4^3 - 3^3) + (5^3 - 4^3)$$

which is the same as

$$(5^3 - 4^3) + (4^3 - 3^3) + (3^3 - 2^3) + (2^3 - 1^3)$$

which simplifies to $5^3 - 1^3$, which is $125 - 1 = 124$. (This type of sum is called a **telescoping** sum.) Use a similar idea to evaluate

$$\sum_{k=5}^{17} (3^k - 3^{k-1}).$$

14. Consider the sum

$$S = \sum_{k=0}^{n} ar^k.$$

Show that $S - rS = a - ar^{n+1}$, which implies that

$$S = \frac{a - ar^{n+1}}{1 - r} \qquad \text{if } r \neq 1.$$

15. Find the value of

$$\sum_{k=1}^{n} f(x_k^*)\ \Delta x_k$$

if

$$f(x) = x + 1,$$
$$a = 0, \qquad b = 4, \qquad n = 3,$$
$$\Delta x_1 = 1, \qquad \Delta x_2 = 1, \qquad \Delta x_3 = 2,$$
$$x_1^* = \frac{1}{3}, \qquad x_2^* = \frac{3}{2}, \qquad x_3^* = 3.$$

It may help to draw a picture of the interval.

16. Find the value of

$$\sum_{k=1}^{n} f(x_k^*)\,\Delta x_k$$

if

$$f(x) = \cos x,$$
$$a = 0, \qquad b = 2\pi, \qquad n = 4,$$
$$\Delta x_1 = \pi/2, \qquad \Delta x_2 = 3\pi/4, \qquad \Delta x_3 = \pi/2, \qquad \Delta x_4 = \pi/4$$
$$x_1^* = \pi/4, \qquad x_2^* = \pi, \qquad x_3^* = 3\pi/2, \qquad x_4^* = 7\pi/4.$$

It may help to draw a picture of the interval.

17. Sketch the region whose signed area is represented by the definite integral, and evaluate the integral using your knowledge of geometry.

(a) $$\int_0^5 2\ dx$$

(b) $$\int_0^\pi \cos x\ dx$$

(c) $$\int_{-1}^2 |2x-3|\ dx$$

(d) $$\int_{-1}^1 \sqrt{1-x^2}\ dx$$

18. If f is the function defined by

$$f(x) = \begin{cases} |x-2|, & \text{if } x \geq 0, \\ x+2, & \text{if } x < 0, \end{cases}$$

then evaluate each of the integrals.

(a) $\displaystyle\int_{-2}^{0} f(x)\,dx$

(b) $\displaystyle\int_{-2}^{2} f(x)\,dx$

(c) $\displaystyle\int_{0}^{6} f(x)\,dx$

(d) $\displaystyle\int_{-4}^{6} f(x)\,dx$

19. Find $\int_{-1}^{2} \big(f(x) + 2g(x)\big)\, dx$ if

$$\int_{-1}^{2} f(x)\, dx = 5 \qquad \text{and} \qquad \int_{-1}^{2} g(x)\, dx = -3.$$

20. Evaluate the integral by completing the square and using your knowledge of geometry.

$$\int_0^{10} \sqrt{10x - x^2}\, dx$$

21. Find the area under the curve $y = x^3$ over the interval $[2, 3]$.

22. Find the area under the curve $y = e^{2x}$ over the interval $[0, \ln 2]$.

23. Find the area under the curve $y = \frac{1}{x}$ over the interval $[1, 5]$.

24. Evaluate the integral.

$$\int_{-2}^{1} (x^2 - 6x + 12)\, dx$$

25. Evaluate the integral.

$$\int_{0}^{\pi/4} \sec^2 \theta \; d\theta$$

26. Evaluate the integral.

$$\int_{-1}^{1} |2x - 1| \; dx$$

27. A particle moves along an s-axis. Use the given information to find the position function of the particle.

(a) $v(t) = 3t^2 - 2t$, $s(0) = 1$

(b) $a(t) = 3\sin 3t$, $v(0) = 3$, $s(0) = 3$.

28. A particle moves with a velocity of $v(t)$ m/s along an s-axis. Find the displacement and the distance traveled by the particle during the given time interval.

(a) $v(t) = \sin t$, $0 \leq t \leq \pi/2$

(b) $v(t) = \cos t$, $\pi/2 \leq t \leq 2\pi$

29. A particle moves with acceleration $a(t)$ m/s^2 along an s-axis and has velocity v_0 m/s at time $t = 0$. Find the displacement and the distance traveled by the particle during the given time interval.

$$a(t) = t - 2, \qquad v_0 = 0, \qquad 1 \leq t \leq 5$$

30. A car traveling 60 mi/h along a straight road decelerates at a constant rate of 11 ft/s^2.

(a) How long will it take until the speed is 45 mi/h?

(b) How far will the car travel before coming to a stop?

31. A car that has stopped at a toll booth leaves the booth with a constant acceleration of 4 ft/s^2. At the time the car leaves the booth it is 2500 ft behind a truck traveling with a constant velocity of 50 ft/s. How long will it take for the car to catch the truck, and how far will the car be from the toll booth at that time?

32. Find the average value of the function over the given interval.

$$f(x) = \sin x, \qquad [0, \pi]$$

33. Find the average value of the function over the given interval.

$$f(x) = \frac{1}{1+x^2}, \qquad [1, \sqrt{3}]$$

34a. Suppose that the velocity function of a particle moving along a coordinate line is $v(t) = 3t^3 + 2$. Find the average velocity of the particle over the time interval $1 \leq t \leq 4$ by integrating.

34b. Suppose that the position function of a particle moving along a coordinate line is $s(t) = 6t^2 + t$. Find the average velocity of the particle over the time interval $1 \leq t \leq 4$ algebraically.

35. Evaluate the definite integral.

$$\int_0^1 (2x+1)^3 \, dx$$

36. Evaluate the definite integral.

$$\int_0^{\pi/6} 2\cos 3x \; dx$$

37. Evaluate the definite integral by expressing it in terms of u and evaluating the resulting integral using your knowledge of geometry.

$$\int_{-5/3}^{5/3} \sqrt{25 - 9x^2}\, dx, \qquad u = 3x$$

38. Find the area under the curve $y = 9/(x+2)^2$ over the interval $[-1, 1]$.

39. Find the average value of $f(x) = x/(5x^2 + 1)^2$ over the interval $[0, 2]$.

40. Find the derivative.

$$\frac{d}{dx}\int_{1}^{x^3} \frac{1}{t}\,dt$$

41a. Find the derivative.

$$\frac{d}{dx}\int_{x}^{\pi} \cos(t^3)\, dt$$

41b. Find the derivative.

$$\frac{d}{dx}\int_{\tan x}^{3}\frac{t^2}{1+t^2}\,dt$$