# MAC2312
# Suggested problems on Chapter 7 material
# (techniques of integration)

Idris Mercer

February 23, 2018

**1.** Evaluate the integral.

$$\int x \sec^2(x^2)\, dx$$

Sub $u = x^2 \Rightarrow du = 2x\, dx$

$\frac{1}{2} du = x\, dx$

Integral $= \int \sec^2(x^2) \cdot x\, dx = \int \sec^2(u) \cdot \frac{1}{2} du$

$= \frac{1}{2} \int \sec^2 u\, du = \frac{1}{2} \tan u + C$

$= \frac{1}{2} \tan(x^2) + C$

**2.** Evaluate the integral.

$$\int \frac{\sin 3x}{2+\cos 3x}\,dx$$

Sub $u = 2 + \cos 3x$

$\Rightarrow du = -3\sin 3x\,dx$

$-\frac{1}{3}du = \sin 3x\,dx$

$$\text{Integral} = \int \frac{1}{2+\cos 3x} \cdot \sin 3x\,dx$$

$$= \int \frac{1}{u} \cdot -\frac{1}{3}\,du$$

$$= -\frac{1}{3}\int \frac{1}{u}\,du$$

$$= -\frac{1}{3}\ln|u| + C$$

$$= -\frac{1}{3}\ln|2+\cos 3x| + C$$

$$\int u\,dv = uv - \int v\,du$$

**3.** Evaluate the integral.

$$\int x^2 e^x\,dx$$

By parts: $\left.\begin{array}{r} u = x^2 \\ dv = e^x dx \end{array}\right\} \Rightarrow \begin{array}{l} du = 2x\,dx \\ v = e^x \end{array}$

Integral $= x^2 e^x - \int e^x \cdot 2x\,dx$

By parts again: $\left.\begin{array}{r} u = 2x \\ dv = e^x dx \end{array}\right\} \Rightarrow \begin{array}{l} du = 2dx \\ v = e^x \end{array}$

Original integral $= x^2 e^x - \left(2x e^x - \int e^x 2dx\right)$

$= x^2 e^x - 2x e^x + 2\int e^x dx$

$= x^2 e^x - 2x e^x + 2e^x + C$

or $(x^2 - 2x + 2)e^x + C$

Could double-check: $\frac{d}{dx}\left((x^2-2x+2)e^x\right) = (2x-2)e^x + (x^2-2x+2)e^x$ ✓

4. Evaluate the integral.

$$\int (\ln x)^2 \, dx$$

Try integration by parts:

$$\left.\begin{aligned} u &= (\ln x)^2 \\ dv &= 1\,dx \end{aligned}\right\} \Rightarrow \begin{aligned} du &= 2(\ln x)\cdot \frac{1}{x}\,dx \\ v &= x \end{aligned}$$

$$\text{Integral} = (\ln x)^2 \cdot x - \int x \cdot 2(\ln x)\cdot \frac{1}{x}\,dx$$

$$= x(\ln x)^2 - 2\int \ln x \, dx$$

Can then do integration by parts again

This time $\left.\begin{aligned} u &= \ln x \\ dv &= 1\,dx \end{aligned}\right\} \Rightarrow \begin{aligned} du &= \frac{1}{x}\,dx \\ v &= x \end{aligned}$

(Also we could maybe remember from before that $\int \ln x \, dx = x \ln x - x$)

Answer: $x(\ln x)^2 - 2(x\ln x - x) + C$

or $x(\ln x)^2 - 2x\ln x + 2x + C$



or $x\left((\ln x)^2 - 2\ln x + 2\right) + C$

**5.** Evaluate the integral.

$$\int e^x \sin x \, dx$$

$$I = \int e^x \sin x \, dx$$

$$\left.\begin{array}{l} u = e^x \\ dv = \sin x \, dx \end{array}\right\} \Rightarrow \begin{array}{l} du = e^x dx \\ v = -\cos x \end{array}$$

$$I = e^x \cdot (-\cos x) - \int (-\cos x) e^x \, dx$$

$$I = -e^x \cos x + \int e^x \cos x \, dx$$

By parts again. Make <u>similar</u> choice ($u = \exp$, $dv = \text{trig}$)

$$\left.\begin{array}{l} u = e^x \\ dv = \cos x \, dx \end{array}\right\} \Rightarrow \begin{array}{l} du = e^x dx \\ v = \sin x \end{array}$$

$$I = -e^x \cos x + e^x \sin x - \int \sin x \cdot e^x \, dx$$

$$I = -e^x \cos x + e^x \sin x - I$$

$$2I = -e^x \cos x + e^x \sin x$$

$$I = \frac{1}{2}\left(-e^x \cos x + e^x \sin x\right) + C$$

or $\frac{1}{2} e^x (\sin x - \cos x) + C$.

Could double-check.
Derivative is
$\frac{1}{2} e^x (\sin x - \cos x) + \frac{1}{2} e^x (\cos x + \sin x)$

6. Evaluate the integral.

$$\int \sin(\ln x)\, dx$$

Sub $u = \ln x \rightsquigarrow e^u = x$

$\Rightarrow du = \frac{1}{x} dx$

$$\text{Integral} = \int x \sin(\ln x) \frac{1}{x} dx$$

$$= \int e^u \sin(u)\, du$$

$$= \frac{1}{2} e^u (\sin u - \cos u) + C \quad \text{by previous problem!}$$

$$= \frac{1}{2} x \left( \sin(\ln x) - \cos(\ln x) \right) + C$$

There may be other methods also

7. Evaluate the integral.

$$\int x^3 e^{x^2}\, dx$$

METHOD 1: Start with substitution. Let $t = x^2$

Then $dt = 2x\,dx$

$\frac{1}{2} dt = x\,dx$

$$I = \int x^2 e^{x^2} x\, dx$$

$$= \int t e^t \cdot \frac{1}{2} dt = \frac{1}{2} \int t e^t\, dt$$

Then integrate by parts: $\left.\begin{array}{l} u = t \\ dv = e^t dt \end{array}\right\} \Rightarrow \begin{array}{l} du = 1\,dt \\ v = e^t \end{array}$

$$I = \frac{1}{2}\left(te^t - \int e^t \cdot 1\,dt\right) = \frac{1}{2}\left(te^t - e^t\right)$$

$$= \frac{1}{2}\left(x^2 e^{x^2} - e^{x^2}\right) + C$$

or $\frac{1}{2}(x^2 - 1)e^{x^2} + C$

METHOD 2: Start with integration by parts, but slightly tricky

$$I = \int \frac{1}{2} x^2 \cdot e^{x^2} \cdot 2x\, dx \qquad \left.\begin{array}{l} u = \frac{1}{2}x^2 \\ dv = e^{x^2} \cdot 2x\,dx \end{array}\right\} \Rightarrow \begin{array}{l} du = x\,dx \\ v = e^{x^2} \end{array}$$

$I = \frac{1}{2} x^2 e^{x^2} - \int e^{x^2} x\, dx$ Then substitute $w = x^2$

$dw = 2x\,dx$

$\frac{1}{2} dw = x\,dx$

$$I = \frac{1}{2} x^2 e^{x^2} - \frac{1}{2} \int e^w\, dw = \frac{1}{2} x^2 e^{x^2} - \frac{1}{2} e^w = \frac{1}{2} x^2 e^{x^2} - \frac{1}{2} e^{x^2} + C$$

**8.** Evaluate the integral.

$$\int_1^e x^3 e^{x^2}\,dx$$

Using result from previous problem, this is

$$\left[\frac{1}{2}(x^2-1)e^{x^2}\right]_{x=1}^{x=e}$$

$$= \frac{1}{2}\left((e^2-1)e^{e^2} - \underbrace{(1^2-1)e^1}_{=0}\right)$$

$$= \frac{1}{2}(e^2-1)e^{e^2}$$

or $\frac{1}{2}\left(e^{2+e^2} - e^{e^2}\right)$

$$\int_{x=a}^{x=b} u\,dv = \Big[uv\Big]_{x=a}^{x=b} - \int_{x=a}^{x=b} v\,du$$

**9.** Evaluate the integral.

$$\int_1^e x^2 \ln x \, dx = I$$

$$\left.\begin{aligned} \text{Let } u &= \ln x \\ dv &= x^2 dx \end{aligned}\right\} \Rightarrow \begin{aligned} du &= \frac{1}{x}\,dx \\ v &= \frac{x^3}{3} \end{aligned}$$

$$I = \left[\ln x \cdot \frac{x^3}{3}\right]_1^e - \int_1^e \frac{x^3}{3} \cdot \frac{1}{x}\,dx$$

$$= \frac{1}{3}\Big(\underbrace{\ln e}_{1} \cdot e^3 - \underbrace{\ln 1}_{0} \cdot 1^3\Big) - \frac{1}{3}\int_1^e x^2\,dx$$

$$= \frac{e^3}{3} - \frac{1}{3}\left[\frac{x^3}{3}\right]_1^e$$

$$= \frac{e^3}{3} - \frac{1}{9}\left(e^3 - 1\right) = \frac{3e^3}{9} - \frac{e^3}{9} + \frac{1}{9}$$

$$= \frac{2e^3 + 1}{9}$$

**10.** Evaluate the integral.
$$\int \sin^2 t \cos^3 t \, dt$$

$$I = \int \sin^2 t \cos^2 t \cos t \, dt$$

$$= \int \sin^2 t \, (1 - \sin^2 t) \cos t \, dt$$

Sub $u = \sin t \Rightarrow du = \cos t \, dt$

$$I = \int u^2 (1 - u^2) \, du = \int (u^2 - u^4) \, du$$

$$= \frac{u^3}{3} - \frac{u^5}{5} + C$$

$$= \frac{\sin^3 t}{3} - \frac{\sin^5 t}{5} + C$$

11. Evaluate the integral.

$$\int \sin^3 x \cos^2 x \, dx$$

$$I = \int \sin^2 x \cos^2 x \sin x \, dx$$

$$= \int (1 - \cos^2 x) \cos^2 x \sin x \, dx$$

Sub $u = \cos x \Rightarrow du = -\sin x \, dx$

$-1 du = \sin x \, dx$

$$I = \int (1 - u^2) u^2 (-1) \, du$$

$$= \int (u^2 - 1) u^2 \, du = \int (u^4 - u^2) \, du$$

$$= \frac{u^5}{5} - \frac{u^3}{3} + C$$

$$= \frac{\cos^5 x}{5} - \frac{\cos^3 x}{3} + C$$

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$$

$$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$$

**12.** Evaluate the integral.

$$\int \sin^2 x \cos^2 x \, dx$$

Fastest way is probably to use trig identities

$$I = \int \frac{1}{2}(1 - \cos 2x) \cdot \frac{1}{2}(1 + \cos 2x)\, dx$$

$$= \frac{1}{4}\int (1 - \cos^2 2x)\, dx = \frac{1}{4}\int \sin^2 2x \, dx$$

$$= \frac{1}{4}\int \frac{1}{2}(1 - \cos 4x)\, dx = \frac{1}{8}\int (1 - \cos 4x)\, dx$$

$$= \frac{1}{8}\left(x - \frac{1}{4}\sin 4x\right) + C$$

or $\frac{1}{8}x - \frac{1}{32}\sin 4x + C$

or $\frac{1}{32}(4x - \sin 4x) + C$

**13.** Evaluate the integral.

$$\int \sin 2x \cos 3x \, dx = I$$

Integration by parts. $\left.\begin{aligned} u &= \sin 2x \\ dv &= \cos 3x\,dx \end{aligned}\right\} \Rightarrow \begin{aligned} du &= 2\cos 2x\,dx \\ v &= \frac{1}{3}\sin 3x \end{aligned}$

$$I = \sin 2x \cdot \frac{1}{3}\sin 3x - \int \frac{1}{3}\sin 3x \cdot 2\cos 2x\,dx$$

$$I = \frac{1}{3}\sin 2x \sin 3x - \frac{2}{3}\int \sin 3x \cos 2x\,dx$$

By parts again. Let $u$ involve $2x$ again.

$$\left.\begin{aligned} u &= \cos 2x \\ dv &= \sin 3x\,dx \end{aligned}\right\} \Rightarrow \begin{aligned} du &= -2\sin 2x\,dx \\ v &= -\frac{1}{3}\cos 3x \end{aligned}$$

$$I = \frac{1}{3}\sin 2x \sin 3x - \frac{2}{3}\left(\cos 2x \cdot \frac{-1}{3}\cos 3x - \int \frac{2}{3}\sin 2x \cos 3x\,dx\right)$$

$$I = \frac{1}{3}\sin 2x \sin 3x + \frac{2}{9}\cos 2x \cos 3x + \frac{4}{9}I$$

$$\frac{5}{9}I = \frac{1}{3}\sin 2x \sin 3x + \frac{2}{9}\cos 2x \cos 3x$$

$$I = \frac{3}{5}\sin 2x \sin 3x + \frac{2}{5}\cos 2x \cos 3x + C$$

Could double-check by taking derivative

**14.** Evaluate the integral.

$$\int_0^{\pi/2} \cos^3 x \, dx = I$$

$$\int_0^{\pi/2} \cos^2 x \cos x \, dx$$

$$= \int_{x=0}^{x=\pi/2} \left(1 - \sin^2 x\right) \cos x \, dx$$

Sub $u = \sin x$ $\quad$ If $x = 0$, then $u = \sin 0 = 0$

$\Rightarrow du = \cos x \, dx$ $\quad$ If $x = \frac{\pi}{2}$, then $u = \sin\frac{\pi}{2} = 1$

$$I = \int_{u=0}^{u=1} \left(1 - u^2\right) du$$

$$= \left[u - \frac{u^3}{3}\right]_{u=0}^{u=1} = \left(1 - \frac{1}{3}\right) - (0 - 0)$$

$$= \frac{2}{3}$$

**15.** Evaluate the integral.

$$\int e^{-x} \tan(e^{-x})\, dx = I$$

Sub $u = e^{-x} \Rightarrow du = -e^{-x}dx$

$$-du = e^{-x}dx$$

$$I = -\int \tan u \, du = -\int \frac{\sin u}{\cos u}\, du$$

Then sub $w = \cos u \Rightarrow dw = -\sin u \, du$

$$I = \int \frac{1}{w}\, dw = \ln|w| + C$$

$$= \ln|\cos u| + C$$

$$= \ln|\cos(e^{-x})| + C$$

**16.** Evaluate the integral.

$$\int \tan^2 x \sec^2 x \, dx = I$$

Sub $u = \tan x \Rightarrow du = \sec^2 x \, dx$

$$I = \int u^2 \, du$$

$$= \frac{u^3}{3} + C$$

$$= \frac{\tan^3 x}{3} + C$$

$$\sin^2 x + \cos^2 x = 1$$
$$\tan^2 x + 1 = \sec^2 x$$
$$\tan^2 x = \sec^2 x - 1$$

17. Evaluate the integral.

$$\int \sec^5 x \tan^3 x \, dx = I$$

$$I = \int \sec^4 x \tan^2 x \cdot \sec x \tan x \, dx$$

$$= \int \sec^4 x \cdot (\sec^2 x - 1) \sec x \tan x \, dx$$

Sub $u = \sec x \Rightarrow du = \sec x \tan x \, dx$

$$I = \int u^4 (u^2 - 1) \, du = \int (u^6 - u^4) \, du$$

$$= \frac{u^7}{7} - \frac{u^5}{5} + C$$

$$= \frac{\sec^7 x}{7} - \frac{\sec^5 x}{5} + C$$

**18.** Evaluate the integral.

$$\int \sqrt{4-x^2}\,dx = I$$

Sub $x = 2\sin\theta$

$dx = 2\cos\theta\, d\theta$

$$I = \int \sqrt{\underbrace{4-4\sin^2\theta}_{\substack{4(1-\sin^2\theta) \\ = 4\cos^2\theta}}}\; 2\cos\theta\, d\theta$$

$$= \int \sqrt{4\cos^2\theta}\; 2\cos\theta\, d\theta = \int 2\cos\theta\; 2\cos\theta\, d\theta = 4\int \cos^2\theta\, d\theta$$

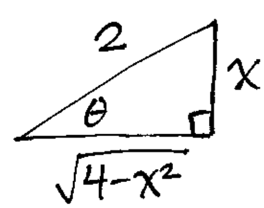
$$= 4\int \frac{1}{2}(1+\cos 2\theta)\, d\theta = 2\int (1+\cos 2\theta)\, d\theta$$

$$= 2\left(\theta + \frac{1}{2}\sin 2\theta\right) + C$$

$$= 2\theta + \sin 2\theta + C$$

Final answer must involve $x$.

$\frac{x}{2} = \sin\theta$




$$= 2\theta + 2\sin\theta\cos\theta + C$$

$$= 2\arcsin\frac{x}{2} + 2\cdot\frac{x}{2}\cdot\frac{\sqrt{4-x^2}}{2} + C$$

**19.** Evaluate the integral.

$$\int \frac{x^2}{\sqrt{16-x^2}}\,dx = I$$

Sub $x = 4\sin\theta \Rightarrow dx = 4\cos\theta\, d\theta$

$$I = \int \frac{16\sin^2\theta}{\sqrt{16-16\sin^2\theta}} 4\cos\theta\, d\theta = \int \frac{16\sin^2\theta}{\sqrt{16\cos^2\theta}} 4\cos\theta\, d\theta$$

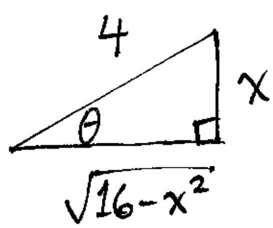
$$= \int \frac{16\sin^2\theta}{4\cos\theta} 4\cos\theta\, d\theta = 16\int \sin^2\theta\, d\theta$$

$$= 16\int \frac{1}{2}\left(1-\cos 2\theta\right) d\theta = 8\int (1-\cos 2\theta)\, d\theta$$

$$= 8\left(\theta - \frac{1}{2}\sin 2\theta\right) + C = 8\theta - 4\sin 2\theta + C$$

$$= 8\theta - 8\sin\theta\cos\theta + C$$

$\frac{x}{4} = \sin\theta$




$$I = 8\arcsin\frac{x}{4} - 8\cdot\frac{x}{4}\cdot\frac{\sqrt{16-x^2}}{4} + C$$

$$= 8\arcsin\frac{x}{4} - \frac{1}{2}x\sqrt{16-x^2} + C$$

**20.** Evaluate the integral.

$$\int \frac{1}{(4+x^2)^2}\,dx = I$$

Sub $x = 2\tan\theta \Rightarrow dx = 2\sec^2\theta\, d\theta$

$$I = \int \frac{1}{(4+4\tan^2\theta)^2}\, 2\sec^2\theta\, d\theta$$

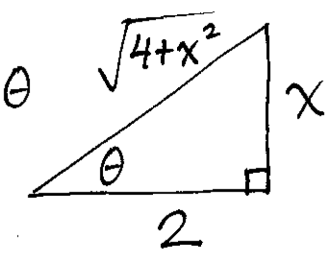
$$= \int \frac{1}{(4\sec^2\theta)^2}\, 2\sec^2\theta\, d\theta = \int \frac{1}{16\sec^4\theta}\, 2\sec^2\theta\, d\theta$$

$$= \frac{1}{8}\int \frac{1}{\sec^2\theta}\, d\theta = \frac{1}{8}\int \cos^2\theta\, d\theta$$

$$= \frac{1}{8}\int \frac{1}{2}(1+\cos 2\theta)\, d\theta = \frac{1}{16}\int (1+\cos 2\theta)\, d\theta$$

$$= \frac{1}{16}\left(\theta + \frac{1}{2}\sin 2\theta\right) + C = \frac{1}{16}\theta + \frac{1}{32}\sin 2\theta + C$$

$$= \frac{1}{16}\theta + \frac{1}{16}\sin\theta\cos\theta + C \qquad \frac{x}{2} = \tan\theta$$




$$= \frac{1}{16}\arctan\frac{x}{2} + \frac{1}{16}\cdot\frac{x}{\sqrt{4+x^2}}\cdot\frac{2}{\sqrt{4+x^2}} = \frac{1}{16}\arctan\frac{x}{2} + \frac{x}{8(4+x^2)}$$

21. Evaluate the integral.

$$\int \frac{\sqrt{x^2-9}}{x}\,dx = I$$

Sub $x = 3\sec\theta \Rightarrow dx = 3\sec\theta\tan\theta\,d\theta$

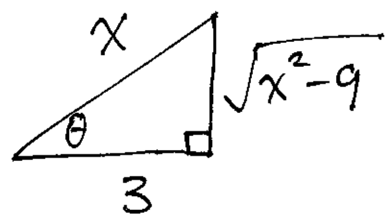
$$I = \int \frac{\sqrt{9\sec^2\theta - 9}}{3\sec\theta}\, 3\sec\theta\tan\theta\,d\theta$$

$$= \int \frac{\sqrt{9\tan^2\theta}}{\cancel{3\sec\theta}}\, \cancel{3\sec\theta}\tan\theta\,d\theta = \int 3\tan\theta\tan\theta\,d\theta$$

$$= 3\int \tan^2\theta\,d\theta = 3\int (\sec^2\theta - 1)\,d\theta$$

$$= 3(\tan\theta - \theta) + C = 3\tan\theta - 3\theta + C$$

$\frac{x}{3} = \sec\theta \qquad \frac{3}{x} = \cos\theta$




$$I = 3\cdot\frac{\sqrt{x^2-9}}{3} - 3\,\text{arcsec}\,\frac{x}{3} + C$$



$$= \sqrt{x^2-9} - 3\,\text{arcsec}\,\frac{x}{3} + C$$

**22.** Evaluate the integral.

$$\int \frac{1}{x^2\sqrt{x^2-16}}\,dx = I$$

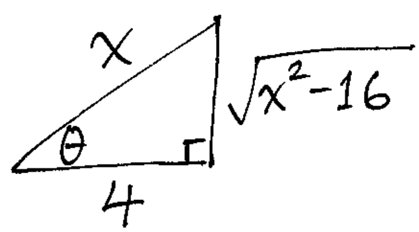
Sub $x = 4\sec\theta \Rightarrow dx = 4\sec\theta\tan\theta\, d\theta$

$$I = \int \frac{1}{16\sec^2\theta\sqrt{\underbrace{16\sec^2\theta - 16}_{16\tan^2\theta}}}\,4\sec\theta\tan\theta\, d\theta$$

$$= \int \frac{1}{16\sec^2\theta \cdot \cancel{4}\cancel{\tan\theta}}\,\cancel{4}\sec\theta\cancel{\tan\theta}\,d\theta = \frac{1}{16}\int \frac{1}{\sec\theta}\,d\theta$$

$$= \frac{1}{16}\int \cos\theta\, d\theta = \frac{1}{16}\sin\theta + C$$

$$\frac{x}{4} = \sec\theta \qquad \frac{4}{x} = \cos\theta$$




$$I = \frac{1}{16}\cdot\frac{\sqrt{x^2-16}}{x} + C = \frac{\sqrt{x^2-16}}{16x} + C$$

**23.** Evaluate the integral.

$$\int \frac{1}{(1-x^2)^{3/2}}\,dx = I$$

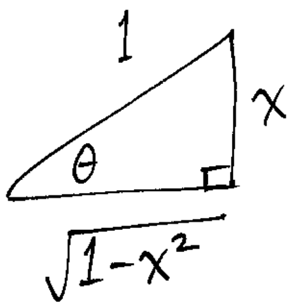
Sub $x = \sin\theta \Rightarrow dx = \cos\theta\, d\theta$

$$I = \int \frac{1}{(1-\sin^2\theta)^{3/2}} \cos\theta\, d\theta$$

$$= \int \frac{1}{(\cos^2\theta)^{3/2}} \cos\theta\, d\theta = \int \frac{1}{\cos^3\theta} \cos\theta\, d\theta$$

$$= \int \frac{1}{\cos^2\theta}\, d\theta = \int \sec^2\theta\, d\theta = \tan\theta + C$$

$$\frac{x}{1} = \sin\theta$$




$$I = \frac{x}{\sqrt{1-x^2}} + C$$

24. Find the arc length of the curve $y = \ln x$ from $x = 1$ to $x = 2$. Tricky problem!

$$y = \ln x \Rightarrow \frac{dy}{dx} = \frac{1}{x}$$

$$\text{Arc length} = \int_{x=1}^{x=2} ds = \int_1^2 \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$= \int_1^2 \sqrt{1 + \frac{1}{x^2}}\, dx = \int_1^2 \sqrt{\frac{x^2+1}{x^2}}\, dx = \int_1^2 \frac{\sqrt{x^2+1}}{x}\, dx = I$$

Sub $x = \tan\theta$ If $x=1$ then $\tan\theta = 1$ so $\theta = \pi/4$

$\Rightarrow dx = \sec^2\theta\, d\theta$. If $x=2$ then $\tan\theta = 2$ so $\theta = \arctan 2$

$$I = \int_{\theta=\pi/4}^{\theta=\arctan 2} \frac{\sqrt{\tan^2\theta + 1}}{\tan\theta} \sec^2\theta\, d\theta = \int_{\pi/4}^{\arctan 2} \frac{\sqrt{\sec^2\theta}}{\tan\theta} \sec^2\theta\, d\theta$$

$$= \int_{\pi/4}^{\arctan 2} \frac{\sec^3\theta}{\tan\theta}\, d\theta = \int_{\pi/4}^{\arctan 2} \frac{\sec^2\theta}{\tan^2\theta} \sec\theta\tan\theta\, d\theta$$

Now sub $u = \sec\theta$ If $\theta = \frac{\pi}{4}$ then $\tan\theta = 1$ so $\sec\theta = \sqrt{2}$

$\Rightarrow du = \sec\theta\tan\theta\, d\theta$ If $\theta = \arctan 2$ then $\tan\theta = 2$ so $\sec\theta = \sqrt{5}$

$\{\sec^2\theta = \tan^2\theta + 1\}$

$$I = \int_{\theta=\pi/4}^{\theta=\arctan 2} \frac{\sec^2\theta}{\sec^2\theta - 1} \sec\theta\tan\theta\, d\theta = \int_{u=\sqrt{2}}^{u=\sqrt{5}} \frac{u^2}{u^2-1}\, du$$

CONTINUED ON NEXT PAGE

$$I = \int_{\sqrt{2}}^{\sqrt{5}} \frac{u^2 - 1 + 1}{u^2 - 1}\, du = \int_{\sqrt{2}}^{\sqrt{5}} \left(1 + \frac{1}{u^2-1}\right) du$$

$$= \int_{\sqrt{2}}^{\sqrt{5}} 1\, du + \int_{\sqrt{2}}^{\sqrt{5}} \frac{1}{u^2-1}\, du = \sqrt{5} - \sqrt{2} + \int_{\sqrt{2}}^{\sqrt{5}} \frac{1}{u^2-1}\, du$$

$$= \sqrt{5} - \sqrt{2} + \frac{1}{2}\int_{\sqrt{2}}^{\sqrt{5}} \frac{2}{u^2-1}\, du = \sqrt{5} - \sqrt{2} + \frac{1}{2}\int_{\sqrt{2}}^{\sqrt{5}} \frac{(u+1)-(u-1)}{u^2-1}\, du$$

$$= \sqrt{5} - \sqrt{2} + \frac{1}{2}\int_{\sqrt{2}}^{\sqrt{5}} \left(\frac{u+1}{u^2-1} - \frac{u-1}{u^2-1}\right) du$$

$$= \sqrt{5} - \sqrt{2} + \frac{1}{2}\int_{\sqrt{2}}^{\sqrt{5}} \left(\frac{1}{u-1} - \frac{1}{u+1}\right) du$$

$$= \sqrt{5} - \sqrt{2} + \frac{1}{2}\Big[\ln|u-1| - \ln|u+1|\Big]_{\sqrt{2}}^{\sqrt{5}}$$

$$= \sqrt{5} - \sqrt{2} + \frac{1}{2}\Big(\ln(\sqrt{5}-1) - \ln(\sqrt{5}+1) - \ln(\sqrt{2}-1) + \ln(\sqrt{2}+1)\Big)$$

**25.** Evaluate the integral.

$$\int \frac{1}{x^2 - 4x + 5}\,dx$$

$$\int \frac{1}{x^2-4x+4+1}\,dx = \int \frac{1}{(x-2)^2+1}\,dx = I$$

Could do trig sub, or just $u = x-2$

$\Rightarrow du = 1\,dx$

$$I = \int \frac{1}{u^2+1}\,du = \arctan u + C$$

$$= \arctan(x-2) + C$$