

WRITE YOUR NAME:

MAC 2312 Homework 1

Due in class, Friday January 19th

You can use more paper if necessary, but please STAPLE

Question 1. Evaluate the sum. You can use theorems from Section 5.4 of the textbook.

$$\sum_{k=0}^{99} (k^2 + 2k + 1)$$

There are several correct ways,
but one correct way is

$$\sum_{k=0}^{99} (k^2 + 2k + 1) = \sum_{k=0}^{99} (k+1)^2$$

$$= \sum_{j=1}^{100} j^2 \quad (j=k+1)$$

$$= \frac{100 \cdot (100+1) \cdot (200+1)}{6}$$

using the formula

$$\sum_{j=1}^n j^2 = \frac{n(n+1)(2n+1)}{6}$$

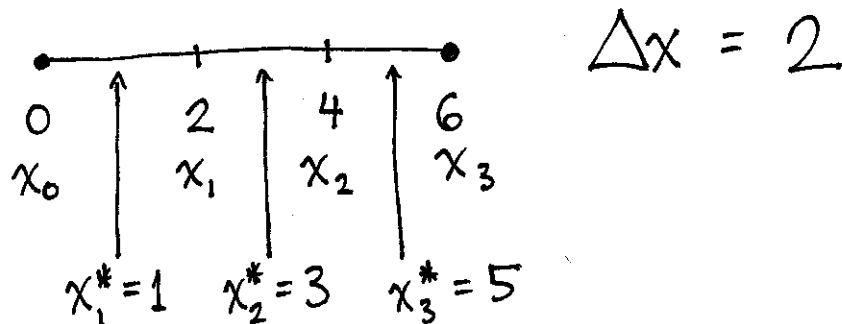
$$= \frac{100 \cdot 101 \cdot 201}{6} = \frac{(2 \cdot 50) \cdot 101 \cdot (3 \cdot 67)}{6}$$

$$= 50 \cdot 101 \cdot 67 = 5050 \cdot 67 = 338350$$

Question 2. Let f be the function $f(x) = \sqrt{x^3 - x + 1}$ on the interval $[0, 6]$. Divide the interval into $n = 3$ subintervals of equal length and then compute

$$\sum_{k=1}^3 f(x_k^*) \Delta x$$

if x_k^* is the midpoint of each subinterval.



$$\begin{aligned} \sum_{k=1}^3 f(x_k^*) \Delta x &= f(x_1^*) \Delta x + f(x_2^*) \Delta x + f(x_3^*) \Delta x \\ &= f(1) \cdot 2 + f(3) \cdot 2 + f(5) \cdot 2 \\ &= 2(f(1) + f(3) + f(5)) \\ &= 2\left(\sqrt{1-1+1} + \sqrt{27-3+1} + \sqrt{125-5+1}\right) \\ &= 2\left(\sqrt{1} + \sqrt{25} + \sqrt{121}\right) = 2(1 + 5 + 11) \\ &= 2 \cdot 17 = 34 \end{aligned}$$