WRITE YOUR NAME:

MAC 2312 Homework 2
Due in class, Friday January 27th
You can use more paper if necessary, but please STAPLE

Question 1. Evaluate the sum.

$$\sum_{j=2}^{6}(3j-1)$$

Because 6 is a small number, we can evaluate directly:

$$(3\cdot 2-1)+(3\cdot 3-1)+(3\cdot 4-1)+(3\cdot 5-1)+(3\cdot 6-1)$$

$$=(6-1)+(9-1)+(12-1)+(15-1)+(18-1)$$

$$=5+8+11+14+17 \quad = 55.$$

Other possible approaches, which can be useful for longer sums:

$$\sum_{j=2}^{6}\big(3j+(-1)\big) = \sum_{j=2}^{6}(3j) + \sum_{j=2}^{6}(-1)$$

$$= 3\sum_{j=2}^{6} j + \sum_{j=2}^{6}(-1)$$

$$= 3(2+3+4+5+6) + 5(-1) \quad = 3(20)-5$$

(five terms in sum)

$$= 55$$

Question 2. Evaluate the sum.

$$\sum_{n=1}^{6} \sin(n\pi)$$

Note that if n is a whole number, then $\sin(n\pi) = 0$.

($\sin \pi$, $\sin 2\pi$, $\sin 3\pi$, ... are all 0)

Answer is $0+0+0+0+0+0 \quad = 0$

By the way, if the question were something like

$$\sum_{n=1}^{6} \sin\left(\frac{n\pi}{12}\right)$$

then there wouldn't be any obvious shortcut.
The answer would just be

$$\sin\left(\frac{\pi}{12}\right) + \sin\left(\frac{2\pi}{12}\right) + \sin\left(\frac{3\pi}{12}\right) + \sin\left(\frac{4\pi}{12}\right) + \sin\left(\frac{5\pi}{12}\right) + \sin\left(\frac{6\pi}{12}\right)$$

We would have to just evaluate those and add them together.

Question 3. Express the sum in sigma notation.

$$a_1 - a_2 + a_3 - a_4 + a_5$$

Possible correct answers include

$$\underbrace{\sum_{k=1}^{5} (-1)^{k+1} a_k}_{\text{this means}} \quad \text{or} \quad \sum_{k=1}^{5} (-1)^{k-1} a_k$$

this means

$$(-1)^2 a_1 + (-1)^3 a_2 + (-1)^4 a_3 + (-1)^5 a_4 + (-1)^6 a_5$$

which is the same as

$$+a_1 - a_2 + a_3 - a_4 + a_5 .$$

$(-1)^k$ or $(-1)^{k+1}$ or $(-1)^{k-1}$ are common tricks for getting something that alternates in sign.

Note: Some other correct answers include

$$\sum_{k=0}^{4} (-1)^k a_{k+1} \quad \text{and} \quad \sum_{k=2}^{6} (-1)^k a_{k-1}$$

Question 4. Evaluate the sum.

$$\sum_{k=1}^{20} k^2$$

We could do this the long way, but realistically, most people would use the help of a computer.

$$1^2 + 2^2 + 3^2 + 4^2 + \cdots + 19^2 + 20^2 = ?$$

FORMULA given in textbook and lecture:

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

Therefore $$\sum_{k=1}^{20} k^2 = \frac{20(20+1)(40+1)}{6}$$

$$= \frac{20 \cdot 21 \cdot 41}{6} = 10 \cdot 7 \cdot 41 = 2870$$

Question 5. Evaluate the sum.

$$\sum_{k=4}^{20} k^2$$

This means $4^2 + 5^2 + \ldots + 19^2 + 20^2$

which is the same as

$$\left(1^2+2^2+3^2+4^2+5^2+\ldots+19^2+20^2\right) - \left(1^2+2^2+3^2\right)$$

or more briefly

$$\sum_{k=1}^{20} k^2 \quad - \quad \sum_{k=1}^{3} k^2$$

this is just $1+4+9=14$

$$= \frac{20(20+1)(40+1)}{6} - \frac{3(3+1)(6+1)}{6}$$

$$= 2870 - 14$$

$$= 2856$$