

WRITE YOUR NAME:

MAC 2312 Homework 2

Due in class, Friday January 26th

You can use more paper if necessary, but please STAPLE

Question 1. Find the area under the graph of $f(x) = x^{-2/3}$ on the interval $[1, 27]$.

$$\begin{aligned}\int_1^{27} x^{-2/3} dx &= \left[\frac{x^{1/3}}{1/3} \right]_1^{27} \\ &= \left[3x^{1/3} \right]_1^{27} = 3 \left[x^{1/3} \right]_1^{27} \\ &= 3 \left(27^{1/3} - 1^{1/3} \right) \\ &= 3 \left(3 - 1 \right) = 3 \cdot 2 = 6\end{aligned}$$

Question 2. Evaluate the integral.

$$\int_{-1}^2 4x(1-x^2) dx$$

$$\int_{-1}^2 (4x - 4x^3) dx$$

$$= \left[4 \frac{x^2}{2} - 4 \frac{x^4}{4} \right]_{-1}^2 = \left[2x^2 - x^4 \right]_{-1}^2$$

$$= \left[2x^2 \right]_{-1}^2 - \left[x^4 \right]_{-1}^2 = \left[2x^2 \right]_{-1}^2 + \left[x^4 \right]_2^{-1}$$

$$= 2 \left[x^2 \right]_{-1}^2 + \left[x^4 \right]_2^{-1}$$

$$= 2 \left(2^2 - (-1)^2 \right) + \left((-1)^4 - 2^4 \right)$$

$$= 2(4-1) + (1-16)$$

$$= 2 \cdot 3 - 15 = 6 - 15 = -9$$

Question 3. Evaluate the integral.

$$\begin{aligned} \int_1^2 x^{-6} dx &= \int_1^2 \frac{1}{x^6} dx = \left[\frac{x^{-5}}{-5} \right]_1^2 \\ &= \left[-\frac{1}{5x^5} \right]_1^2 = \left[\frac{1}{5x^5} \right]_2^1 \\ &= \frac{1}{5} \left[\frac{1}{x^5} \right]_2^1 = \frac{1}{5} \left(\frac{1}{1^5} - \frac{1}{2^5} \right) \\ &= \frac{1}{5} \left(1 - \frac{1}{32} \right) = \frac{1}{5} \cdot \frac{31}{32} = \frac{31}{160} \end{aligned}$$

Question 4. Evaluate the integral.

$$\int_{1/2}^1 \frac{1}{2x} dx$$

$$\frac{1}{2} \int_{1/2}^1 \frac{1}{x} dx = \frac{1}{2} \left[\ln|x| \right]_{1/2}^1$$

$$= \frac{1}{2} \left(\ln|1| - \ln\left|\frac{1}{2}\right| \right)$$

$$= \frac{1}{2} \left(\underbrace{\ln 1}_0 - \ln\left(\frac{1}{2}\right) \right)$$

$$= -\frac{1}{2} \ln\left(\frac{1}{2}\right)$$

Using rules of logs, this is the same as $\frac{1}{2} \ln 2$

$$\ln\left(\frac{1}{a}\right) = \underbrace{\ln 1}_0 - \ln a = -\ln a$$

$$\begin{aligned} \ln\left(\frac{1}{2}\right) &= -\ln 2 \\ -\ln\left(\frac{1}{2}\right) &= \ln 2 \end{aligned}$$