WRITE YOUR NAME:

MAC 2312 Homework 3
Due in class, Friday February 17th
You can use more paper if necessary, but please STAPLE

Question 1. Evaluate the integral.

$$\int_{\pi/12}^{\pi/9} \sec^2 3x \, dx$$

Sub $u = 3x \Rightarrow \frac{du}{dx} = 3 \Rightarrow du = 3dx$
$\Rightarrow \frac{1}{3} du = dx$

If $x = \frac{\pi}{12}$, then $u = 3 \cdot \frac{\pi}{12} = \frac{\pi}{4}$

If $x = \frac{\pi}{9}$, then $u = 3 \cdot \frac{\pi}{9} = \frac{\pi}{3}$

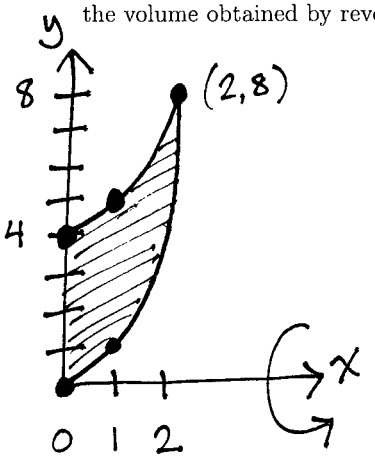
Old integral $= \int_{x=\pi/12}^{x=\pi/9} \sec^2 3x \cdot dx$

$= \int_{u=\pi/4}^{u=\pi/3} \sec^2 u \cdot \frac{1}{3} du$

$= \frac{1}{3} \int_{\pi/4}^{\pi/3} \sec^2 u \, du = \frac{1}{3} \left[\tan u \right]_{\pi/4}^{\pi/3}$

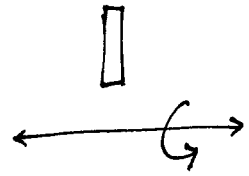
$= \frac{1}{3} \left(\tan \frac{\pi}{3} - \tan \frac{\pi}{4} \right) = \frac{1}{3} \left(\sqrt{3} - 1 \right)$

Question 2. Notice that the curves $y = x^2 + 4$ and $y = x^3$ intersect when $x = 2$. Let A be the region bounded by those two curves and the y-axis. Find the volume obtained by revolving the region A around the x-axis.



$y = f(x) \Rightarrow$ ▯ $\Rightarrow \int \ldots dx$

$\leftrightarrow$ dx



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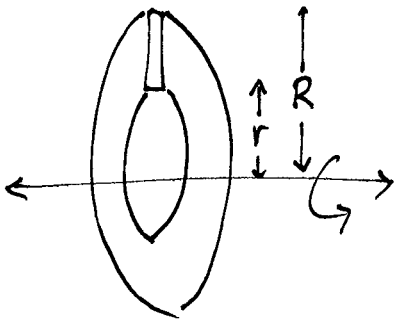

$R = (\text{top curve}) - (\text{axis of revolution})$

$\{y = x^2+4\}$ $\{y = 0\}$

$R = (x^2+4) - 0 = x^2 + 4$

$r = (\text{bottom curve}) - (\text{axis of revolution})$

$\{y = x^3\}$ $\{y = 0\}$

$r = x^3 - 0 = x^3$

$$\text{Volume} = \int_{x=0}^{x=2} (\pi R^2 - \pi r^2)\,dx = \pi \int_0^2 (R^2 - r^2)\,dx$$

$$= \pi \int_0^2 \left((x^2+4)^2 - (x^3)^2 \right) dx = \pi \int_0^2 \left(x^4 + 8x^2 + 16 - x^6 \right) dx$$

$$= \pi \left[\frac{x^5}{5} + \frac{8x^3}{3} + 16x - \frac{x^7}{7} \right]_0^2$$

$$= \pi \left(\frac{2^5}{5} + \frac{8 \cdot 2^3}{3} + 16 \cdot 2 - \frac{2^7}{7} \right)$$

I am OK with stopping there. Simplifies to $\frac{4352}{105}\pi$