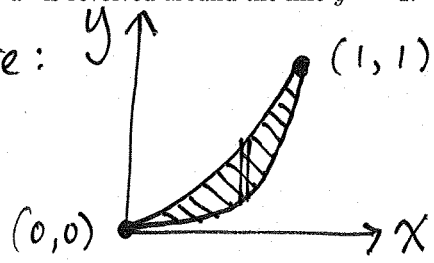
WRITE YOUR NAME:

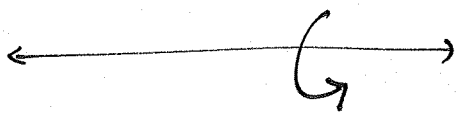
MAC 2312 Homework 4
Due in class, Friday February 24th
You can use more paper if necessary, but please STAPLE

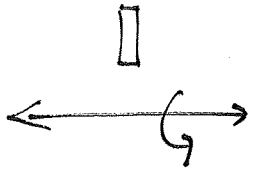
Question 1. Find the volume of the solid that results when the region enclosed by $y = x^2$ and $y = x^3$ is revolved around the line $y = -1$.

Rough picture:



$y = x^2$ is top
$y = x^3$ is bottom





WASHERS

R = top curve − axis of revolution
$= x^2 - (-1) = x^2 + 1$

r = bottom curve − axis of revolution
$= x^3 - (-1) = x^3 + 1$

$$\text{Volume} = \int_0^1 (\pi R^2 - \pi r^2)\,dx = \pi \int_0^1 \left((x^2+1)^2 - (x^3+1)^2\right) dx$$

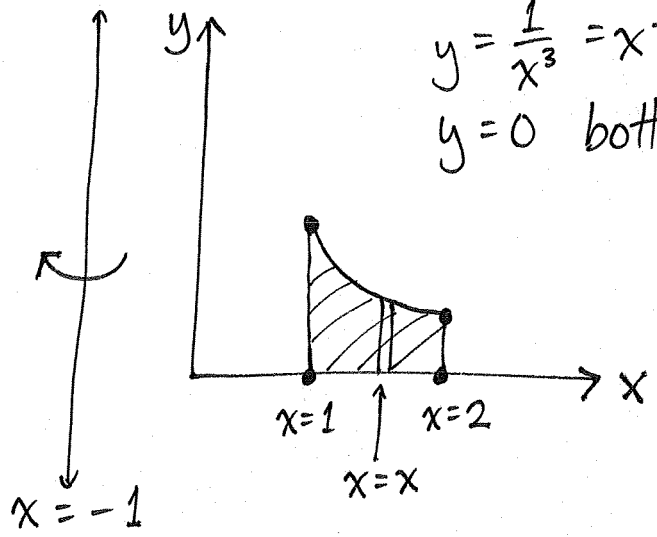
$$= \pi \int_0^1 \left(x^4 + 2x^2 + 1 - (x^6 + 2x^3 + 1)\right) dx = \pi \int_0^1 \left(x^4 + 2x^2 - x^6 - 2x^3\right) dx$$

$$= \pi \left[\frac{x^5}{5} + \frac{2x^3}{3} - \frac{x^7}{7} - \frac{2x^4}{4} \right]_0^1 = \pi \left(\frac{1}{5} + \frac{2}{3} - \frac{1}{7} - \frac{1}{2} \right)$$

$$= \pi \left(\frac{42}{210} + \frac{140}{210} - \frac{30}{210} - \frac{105}{210} \right) = \frac{47\pi}{210}$$

Question 2. Let A be the region that is enclosed by $y = 1/x^3$, $x = 1$, $x = 2$, and $y = 0$. Find the volume of the solid that is generated when the region A is revolved around the line $x = -1$.

Rough picture:



$y = \frac{1}{x^3} = x^{-3}$ top

$y = 0$ bottom

SHELLS r = slice-axis $= x-(-1) = x+1$

h = top curve − bottom curve $= x^{-3} - 0 = x^{-3}$

$$\text{Volume} = \int_1^2 2\pi r h\,dx = 2\pi \int_1^2 (x+1)x^{-3}\,dx$$

$$= 2\pi \int_1^2 (x^{-2} + x^{-3})\,dx = 2\pi \left[\frac{x^{-1}}{-1} + \frac{x^{-2}}{-2} \right]_1^2$$

$$= 2\pi \left[-\frac{1}{x} - \frac{1}{2x^2} \right]_1^2 = 2\pi \left[\frac{1}{x} + \frac{1}{2x^2} \right]_2^1$$

$$= 2\pi \left(\left(1 + \frac{1}{2}\right) - \left(\frac{1}{2} + \frac{1}{8}\right) \right) = 2\pi \left(1 - \frac{1}{8}\right)$$

$$= 2\pi \cdot \frac{7}{8} = \frac{7\pi}{4}$$