WRITE YOUR NAME:

MAC 2312 Homework 5
Due in class, Friday March 24th
You can use more paper if necessary, but please STAPLE

Question 1. Evaluate the integral.

$$\int_0^2 xe^{2x}\,dx$$

Looks like integration by parts. x is easy to take derivative of
e^{2x} is easy to take antiderivative of.

$$\int_{x=a}^{x=b} u\,dv = \Big[uv\Big]_{x=a}^{x=b} - \int_{x=a}^{x=b} v\,du$$

Let $\left.\begin{aligned} u &= x \\ dv &= e^{2x}\,dx \end{aligned}\right\} \Rightarrow \begin{aligned} du &= 1\,dx \\ v &= \tfrac{1}{2}e^{2x} \end{aligned}$

$$\text{Integral} = \left[x\cdot\frac{1}{2}e^{2x}\right]_0^2 - \int_0^2 \frac{1}{2}e^{2x}\cdot 1\,dx$$

$$= 2\cdot\frac{1}{2}e^4 - 0 - \frac{1}{2}\int_0^2 e^{2x}\,dx$$

$$= e^4 - \frac{1}{2}\left[\frac{1}{2}e^{2x}\right]_0^2 = e^4 - \frac{1}{4}\left[e^{2x}\right]_0^2$$

$$= e^4 - \frac{1}{4}\left(e^4 - e^0\right) = e^4 - \frac{1}{4}\left(e^4 - 1\right)$$

$$\text{or } \frac{3}{4}e^4 + \frac{1}{4} \quad \text{or} \quad \frac{3e^4+1}{4}$$

Question 2. Evaluate the integral.

$$\int \cos^{1/3} x \sin x \, dx$$

Looks like u-substitution. $(\cos x)^{1/3}$ has $\cos x$ as "inside"

Let $u = \cos x$

Then $du = -\sin x \, dx$ which more or less appears in integral

$-du = \sin x \, dx$

Integral $\int \underbrace{(\cos x)^{1/3}}_{u^{1/3}} \underbrace{\sin x \, dx}_{-du} = \int u^{1/3} \cdot (-1) \, du$

$$= -\int u^{1/3} \, du = -\frac{u^{4/3}}{4/3} + C$$

$$= -\frac{3}{4} u^{4/3} + C = -\frac{3}{4} (\cos x)^{4/3} + C$$

One possible strategy: Remember $\cos 2x = \cos^2 x - \sin^2 x$

$\cos 2x = \cos^2 x - (1-\cos^2 x) = 2\cos^2 x - 1 \Rightarrow 1+\cos 2x = 2\cos^2 x$

$\cos 2x = (1-\sin^2 x) - \sin^2 x = 1 - 2\sin^2 x \Rightarrow 1-\cos 2x = 2\sin^2 x$

Question 3. Evaluate the integral.

$$\int_0^{\pi/2} \sin^2 x \, dx$$

Remember that to integrate $\sin^2 x$ or $\cos^2 x$, you need* to use a trig identity.
(* there could be other ways too)

$\sin^2 x = \frac{1}{2}(1-\cos 2x)$

$\cos^2 x = \frac{1}{2}(1+\cos 2x)$

$$I = \int_0^{\pi/2} \sin^2 x \, dx = \int_0^{\pi/2} \frac{1}{2}(1-\cos 2x)\, dx$$

$$= \frac{1}{2}\int_0^{\pi/2} (1-\cos 2x)\, dx$$

Sub $u = 2x$

$\Rightarrow du = 2dx \Rightarrow \frac{1}{2}du = dx$

$x = 0 \Rightarrow u = 0$

$x = \frac{\pi}{2} \Rightarrow u = \pi$

$$I = \frac{1}{2}\int_{u=0}^{u=\pi} (1-\cos u)\frac{1}{2}\, du = \frac{1}{4}\int_0^{\pi} (1-\cos u)\, du$$

$$= \frac{1}{4}\Big[u - \sin u\Big]_0^{\pi} = \frac{1}{4}\Big((\pi - \underbrace{\sin\pi}_{0}) - (0 - \underbrace{\sin 0}_{0})\Big) = \frac{\pi}{4}.$$

Note: You can also use the shortcut that $\int \cos 2x \, dx = \frac{1}{2}\sin 2x + c$

Question 4. Evaluate the integral.

$$\int \tan^2 x \sec^2 x \, dx$$

There may be other correct methods, but one possibility is to notice tan x inside another function, and the derivative of tan x.

Try substitution $u = \tan x$
$du = \sec^2 x \, dx$

$$\text{Integral} = \int \underbrace{\tan^2 x}_{u^2} \underbrace{\sec^2 x \, dx}_{du} = \int u^2 \, du$$

$$= \frac{u^3}{3} + C = \frac{(\tan x)^3}{3} + C \text{ or } \frac{\tan^3 x}{3} + C.$$

If necessary: $\sin^2 x + \cos^2 x = 1 \Rightarrow \frac{\sin^2 x}{\cos^2 x} + \frac{\cos^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$

$$\tan^2 x + 1 = \sec^2 x$$

Question 5. Evaluate the integral.

$$\int \tan^5 x \sec^4 x \, dx$$

Guidelines to remember:

- Think about candidates for du
- Remember that even powers of trig functions are easy to rewrite.

Maybe $u = \tan x$? $\Rightarrow du = \sec^2 x \, dx$

$$\text{Integral} = \int \underbrace{\tan^5 x}_{u^5} \underbrace{\sec^2 x}_{\text{easy to rewrite}} \underbrace{\sec^2 x \, dx}_{du}$$

$$= \int \underbrace{\tan^5 x}_{u^5} \underbrace{(\tan^2 x + 1)}_{u^2+1} \underbrace{\sec^2 x \, dx}_{du} = \int u^5 (u^2 + 1) \, du$$

$$= \int (u^7 + u^5) \, du = \frac{u^8}{8} + \frac{u^6}{6} + C$$

$$= \frac{\tan^8 x}{8} + \frac{\tan^6 x}{6} + C$$

Another possible approach: Try $u = \sec x \Rightarrow du = \sec x \tan x \, dx$

$$\text{Integral} = \int \underbrace{\tan^4 x}_{\substack{(\tan^2 x)^2 \\ \text{Easy to rewrite}}} \underbrace{\sec^3 x}_{u^3} \underbrace{\sec x \tan x \, dx}_{du}$$

Remember $\sin^2\theta + \cos^2\theta = 1$

divide by $\cos^2\theta$

$\tan^2\theta + 1 = \sec^2\theta$

θ	$\sin\theta$	$\cos\theta$
0	0	1
$\pi/6$	$1/2$	$\sqrt{3}/2$
$\pi/4$	$1/\sqrt{2}$	$1/\sqrt{2}$
$\pi/3$	$\sqrt{3}/2$	$1/2$
$\pi/2$	1	0

Question 6. Evaluate the integral.

$$\int_{\sqrt{2}}^{2} \frac{1}{x^2\sqrt{x^2-1}}\,dx$$

Looks like trig substitution. Since $\sec^2\theta - 1$ is a perfect square,

try $x = \sec\theta \Rightarrow dx = \sec\theta\tan\theta\,d\theta$

$x = \sqrt{2} \Rightarrow \sec\theta = \sqrt{2} \Rightarrow \cos\theta = \frac{1}{\sqrt{2}} \Rightarrow \theta = \frac{\pi}{4}$

$x = 2 \Rightarrow \sec\theta = 2 \Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$

$$\text{Integral} = \int_{\theta=\pi/4}^{\theta=\pi/3} \frac{1}{\sec^2\theta\sqrt{\sec^2\theta - 1}} \sec\theta\tan\theta\,d\theta$$

$$= \int_{\pi/4}^{\pi/3} \frac{1}{\sec^2\theta\sqrt{\tan^2\theta}} \sec\theta\tan\theta\,d\theta = \int_{\pi/4}^{\pi/3} \frac{1}{\sec^2\theta\tan\theta} \sec\theta\tan\theta\,d\theta$$

$$= \int_{\pi/4}^{\pi/3} \frac{1}{\sec\theta}\,d\theta = \int_{\pi/4}^{\pi/3} \cos\theta\,d\theta = \Big[\sin\theta\Big]_{\pi/4}^{\pi/3}$$

$$= \sin\frac{\pi}{3} - \sin\frac{\pi}{4} = \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} = \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}$$

$$= \frac{\sqrt{3}-\sqrt{2}}{2}$$ or about $\frac{1.732 - 1.414}{2}$

$= \frac{0.318}{2} = 0.159$

Recall: $\int u\,dv = uv - \int v\,du$

$$\int_{x=a}^{x=b} u\,dv = \Big[uv\Big]_{x=a}^{x=b} - \int_{x=a}^{x=b} v\,du$$

Question 3. Evaluate the integral.

$$\int_0^{\pi/2} \sin^2 x\,dx$$

ALTERNATIVE way to evaluate $\int \sin^2 x\,dx$ or $\int_a^b \sin^2 x\,dx$

$$I = \int \sin^2 x\,dx = \int \sin x \cdot \sin x\,dx$$

By parts: $\left.\begin{aligned} u &= \sin x \\ dv &= \sin x\,dx \end{aligned}\right\} \Rightarrow \begin{aligned} du &= \cos x\,dx \\ v &= -\cos x \end{aligned}$

$$I = \sin x \cdot (-\cos x) - \int (-\cos x)\cos x\,dx$$

$$I = -\sin x \cos x + \int \cos^2 x\,dx$$

$$I = -\sin x \cos x + \int (1 - \sin^2 x)\,dx$$

$$I = -\sin x \cos x + \int 1\,dx - \int \sin^2 x\,dx$$

$$I = -\sin x \cos x + x - I$$

$$2I = -\sin x \cos x + x = x - \sin x \cos x$$

$$I = \frac{1}{2}(x - \sin x \cos x) + C$$

ALTERNATIVE method: Integration by parts

Question 4. Evaluate the integral.

$$\int \tan^2 x \sec^2 x \, dx$$

$$I = \int \tan^2 x \sec^2 x \, dx$$

$$\left.\begin{aligned} u &= \tan^2 x \\ dv &= \sec^2 x \, dx \end{aligned}\right\} \Rightarrow \begin{aligned} du &= 2\tan x \cdot \sec^2 x \, dx \\ v &= \tan x \end{aligned}$$

$$I = \tan^2 x \cdot \tan x - \int \tan x \cdot 2\tan x \sec^2 x \, dx$$

$$I = \tan^3 x - 2\int \tan^2 x \sec^2 x \, dx$$

$$I = \tan^3 x - 2I$$

$$3I = \tan^3 x$$

$$I = \frac{1}{3}\tan^3 x + C$$