WRITE YOUR NAME:

MAC 2312 WRITTEN HOMEWORK #1

Due Tuesday January 16th, in Canvas

Question 1.
Evaluate the midpoint Riemann sum for the function $f(x) = \sin x$ on the interval $[0, \pi]$ using $n = 3$ subintervals.

$$\Delta x = \frac{b-a}{n} = \frac{\pi - 0}{3} = \frac{\pi}{3}$$

Grid points: $0, 0+\frac{\pi}{3}, 0+\frac{2\pi}{3}, 0+\frac{3\pi}{3} = \underset{\substack{\uparrow \\ a = x_0}}{0}, \underset{\substack{\uparrow \\ x_1}}{\frac{\pi}{3}}, \underset{\substack{\uparrow \\ x_2}}{\frac{2\pi}{3}}, \underset{\substack{\uparrow \\ x_3 = b}}{\pi}$

(For finding midpoints, note that the gridpoints are also $0, \frac{2\pi}{6}, \frac{4\pi}{6}, \frac{6\pi}{6}$)

Midpoints of subintervals: $x_1^* = \frac{\pi}{6}$, $x_2^* = \frac{3\pi}{6} = \frac{\pi}{2}$, $x_3^* = \frac{5\pi}{6}$

$$\text{Riemann sum} = f(x_1^*)\Delta x + f(x_2^*)\Delta x + f(x_3^*)\Delta x$$

$$= \underbrace{\sin(\pi/6)}_{1/2} \cdot \frac{\pi}{3} + \underbrace{\sin(\pi/2)}_{1} \cdot \frac{\pi}{3} + \underbrace{\sin(5\pi/6)}_{1/2} \cdot \frac{\pi}{3}$$

$$= \underbrace{\left(\frac{1}{2} + 1 + \frac{1}{2}\right)}_{2} \cdot \frac{\pi}{3} = \boxed{\frac{2\pi}{3}}$$ (approximately 2.09)

By the way, the true value of the integral is $\int_0^{\pi} \sin x \, dx$

$$= [-\cos x]_0^{\pi} = [\cos x]_{\pi}^{0} = \underbrace{\cos 0}_{1} - \underbrace{\cos \pi}_{-1} = 1 - (-1) = 1 + 1 = 2$$

Question 2.
Evaluate the sum.

$$\sum_{k=1}^{5}(100k^2 + 11)$$

$$\sum_{k=1}^{5}(100k^2) + \underbrace{\sum_{k=1}^{5} 11}$$

Note that this is $\underset{k=1}{11} + \underset{k=2}{11} + \underset{k=3}{11} + \underset{k=4}{11} + \underset{k=5}{11}$

$$= 100\sum_{k=1}^{5} k^2 + 5\cdot 11$$

$$= 100(1^2 + 2^2 + 3^2 + 4^2 + 5^2) + 55$$

$$= 100(1 + 4 + 9 + 16 + 25) + 55$$

$$= 100(55) + 55 = 5500 + 55 = \boxed{5555}$$

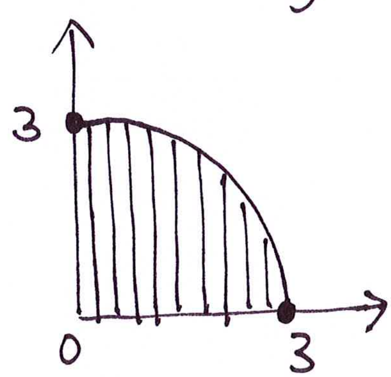
If you like, you can also use the formula $\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$

We have $\sum_{k=1}^{5} k^2 = \frac{5(5+1)(2\cdot 5+1)}{6} = \frac{5\cdot 6\cdot 11}{6} = 55$

Question 3.
Evaluate the definite integral using your knowledge of geometry.

$$\int_0^3 \sqrt{9-x^2}\,dx$$

If $y=\sqrt{9-x^2}$ then $y^2=9-x^2 \Rightarrow x^2+y^2=9$ (radius 3)
Graph is a portion of a circle. y is positive because of $\sqrt{\ }$
and we're only using x between 0 and 3.



$$\text{Integral} = \frac{1}{4} \text{ of area of circle w. radius } 3$$

$$= \frac{1}{4}\cdot\pi(3)^2$$

$$= \boxed{\frac{9\pi}{4}}$$

Question 4.
Evaluate the right-endpoint Riemann sum for the function $f(x) = x^2$ on the interval $[0, 6]$ using $n = 60$ subintervals.

$$\Delta x = \frac{b-a}{n} = \frac{6-0}{60} = \frac{6}{60} = \frac{1}{10} \text{ or } 0.1$$

The 61 grid points are $a, a+\Delta x, a+2\Delta x, a+3\Delta x, \ldots, b$

i.e. $x_0 = 0,\ x_1 = 0.1,\ x_2 = 0.2,\ x_3 = 0.3, \ldots, x_{59} = 5.9,\ x_{60} = 6$

In general, $x_k = a + k\Delta x = 0 + k \cdot \frac{1}{10} = \frac{k}{10}$.

For a right-endpoint Riemann sum, we have $x_k^* = x_k$.

$$\sum_{k=1}^{60} f(x_k^*)\Delta x = \sum_{k=1}^{60} f(x_k)\Delta x = \sum_{k=1}^{60} f\left(\frac{k}{10}\right) \cdot \frac{1}{10}$$

$$= \sum_{k=1}^{60} \left(\frac{k}{10}\right)^2 \cdot \frac{1}{10} = \sum_{k=1}^{60} \frac{k^2}{100} \cdot \frac{1}{10} = \frac{1}{1000} \sum_{k=1}^{60} k^2$$

$$= \frac{1}{1000} \cdot \frac{60(60+1)(2 \cdot 60+1)}{6} = \frac{1}{1000} \cdot \frac{60 \cdot 61 \cdot 121}{6}$$

$$= \frac{1}{1000} \cdot 10 \cdot 61 \cdot 121 = \frac{61 \cdot 121}{100} = \boxed{73.81}$$