WRITE YOUR NAME:

MAC 2312 WRITTEN HOMEWORK #2

Due Tuesday January 23rd, in Canvas

Question 1. Evaluate the integral.

$$\int \cos 2x \, dx$$

Substitute $u = 2x \Rightarrow \frac{du}{dx} = 2 \Rightarrow du = 2dx$

$\Rightarrow \frac{1}{2} du = dx$

$$\int \cos(\underbrace{2x}_{u}) \cdot \underbrace{dx}_{\frac{1}{2}du} = \int \cos u \cdot \frac{1}{2} \, du$$

$$= \frac{1}{2} \int \cos u \, du = \frac{1}{2} \sin u + C$$

$$= \frac{1}{2} \sin 2x + C$$

Double-check: $\frac{d}{dx}\left(\frac{1}{2}\sin(\underbrace{2x}_{\text{INSIDE}})\right) = \frac{1}{2} \cdot \cos(2x) \cdot \underbrace{2}_{\text{DERIV. OF INSIDE}}$

$= \cos(2x)$ ✓

Question 2. Given the following trigonometric identities

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x) \qquad \cos^2 x = \frac{1}{2}(1 + \cos 2x)$$

use them to evaluate both of the following integrals.

$$\int_0^{\pi/2} \sin^2 x \, dx$$

$$\int_0^{\pi/2} \frac{1}{2}(1 - \cos 2x)\,dx = \frac{1}{2}\int_0^{\pi/2} (1 - \cos 2x)\,dx$$

$$= \frac{1}{2}\left[x - \frac{1}{2}\sin 2x\right]_0^{\pi/2} = \frac{1}{2}\left[x\right]_0^{\pi/2} - \frac{1}{4}\left[\sin 2x\right]_0^{\pi/2}$$

$$= \frac{1}{2}\left(\frac{\pi}{2} - 0\right) - \frac{1}{4}\left(\underbrace{\sin \pi}_{0} - \underbrace{\sin 0}_{0}\right) = \boxed{\frac{\pi}{4}}$$

$$\int_0^{\pi/2} \cos^2 x \, dx$$

$$\int_0^{\pi/2} \frac{1}{2}(1 + \cos 2x)\,dx = \frac{1}{2}\int_0^{\pi/2} (1 + \cos 2x)\,dx$$

$$= \frac{1}{2}\left[x + \frac{1}{2}\sin 2x\right]_0^{\pi/2} = \frac{1}{2}\left[x\right]_0^{\pi/2} + \frac{1}{4}\left[\sin 2x\right]_0^{\pi/2}$$

$$= \frac{1}{2}\left(\frac{\pi}{2} - 0\right) + \frac{1}{4}\left(\underbrace{\sin \pi}_{0} - \underbrace{\sin 0}_{0}\right) = \boxed{\frac{\pi}{4}}$$

Question 3. Evaluate the integral.

$$\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$$

Try substituting $u = \sqrt{x} = x^{1/2}$

Then $\frac{du}{dx} = \frac{1}{2}x^{-1/2} \Rightarrow du = \frac{1}{2}x^{-1/2}dx = \frac{1}{2}\cdot\frac{1}{\sqrt{x}}\,dx$

$\Rightarrow 2du = \frac{1}{\sqrt{x}}\,dx$

Integral $= \int e^{\sqrt{x}} \cdot \underbrace{\frac{1}{\sqrt{x}}\,dx}_{2du} = \int e^u \, 2du$

$= 2\int e^u\,du = 2e^u + C$

$= \boxed{2e^{\sqrt{x}} + C}$

Question 4. Evaluate the integral.

$$\int_0^3 \frac{2x-1}{x+1}\,dx$$

Try substituting $u = x+1 \Rightarrow \frac{du}{dx} = 1 \Rightarrow du = dx$

If $x=0$ then $u = 0+1 = 1$

If $x=3$ then $u = 3+1 = 4$

$u = x+1 \Rightarrow u-1 = x$

$$\int_{x=0}^{x=3} \frac{2x-1}{x+1}\,dx = \int_{u=1}^{u=4} \frac{2(u-1)-1}{u}\,du$$

$$= \int_{u=1}^{u=4} \frac{2u-3}{u}\,du = \int_{u=1}^{u=4} \left(2 - \frac{3}{u}\right) du$$

$$= \Big[\, 2u - 3\ln|u| \,\Big]_{u=1}^{u=4} = 2\big[u\big]_{u=1}^{u=4} - 3\big[\ln|u|\big]_{u=1}^{u=4}$$

$$= 2(\underbrace{4-1}_{3}) - 3(\ln 4 - \underbrace{\ln 1}_{0})$$

$$= 6 - 3\ln 4 \quad \text{OR} \quad 6 - 3\ln(2^2) = 6 - 6\ln 2$$