WRITE YOUR NAME:

MAC 2312 WRITTEN HOMEWORK #5

Due Tuesday February 13th, in Canvas

Question 1. Evaluate the integral.

$$\int \tan^3 x \sec^3 x \, dx$$

$$\int \underbrace{\tan^2 x \sec^2 x}_{\text{Even powers}} \cdot \underbrace{\sec x \tan x \, dx}_{\text{"candidate" for } du}$$

Even powers
Easy to rewrite
Remember $\sin^2 x + \cos^2 x = 1$
$\tan^2 x + 1 = \sec^2 x$

Sub $u = \sec x$

$\downarrow$ AUTOMATIC REFLEX

$du = \sec x \tan x \, dx$

$\dfrac{du}{dx} = \sec x \tan x$

$$\int \underbrace{(\sec^2 x - 1)}_{u^2 - 1} \underbrace{\sec^2 x}_{u^2} \cdot \underbrace{\sec x \tan x \, dx}_{du}$$

$$= \int (u^2 - 1) u^2 \, du = \int (u^4 - u^2) \, du$$

$$= \frac{u^5}{5} - \frac{u^3}{3} + C = \frac{\sec^5 x}{5} - \frac{\sec^3 x}{3} + C$$

Question 2. Evaluate the integral. Check your answer by differentiating.

$$\int e^{2x} \cos 5x \, dx$$

$$I = \int \underbrace{e^{2x}}_{u} \underbrace{\cos 5x \, dx}_{dv} = \underbrace{e^{2x}}_{u} \cdot \underbrace{\frac{1}{5} \sin 5x}_{v} - \int \underbrace{\frac{1}{5} \sin 5x}_{v} \cdot \underbrace{2e^{2x} dx}_{du}$$

$$u = e^{2x} \Rightarrow du = 2e^{2x} dx$$

$$dv = \cos 5x \, dx \Rightarrow v = \frac{1}{5} \sin 5x$$

$$I = \frac{1}{5} e^{2x} \sin 5x - \frac{2}{5} \int \underbrace{e^{2x}}_{} \underbrace{\sin 5x \, dx}_{}$$

Integrate by parts <u>again</u>. This time $u = e^{2x} \Rightarrow du = 2e^{2x} dx$

$$dv = \sin 5x \, dx \Rightarrow v = -\frac{1}{5} \cos 5x$$

$$I = \frac{1}{5} e^{2x} \sin 5x - \frac{2}{5} \left(\underbrace{e^{2x}}_{u} \cdot \underbrace{\frac{-1}{5} \cos 5x}_{v} - \int \underbrace{\frac{-1}{5} \cos 5x}_{v} \cdot \underbrace{2e^{2x} dx}_{du} \right)$$

$$I = \frac{1}{5} e^{2x} \sin 5x - \frac{2}{5} \left(-\frac{1}{5} e^{2x} \cos 5x + \frac{2}{5} \int e^{2x} \cos 5x \, dx \right)$$

$$I = \frac{1}{5} e^{2x} \sin 5x + \frac{2}{25} e^{2x} \cos 5x - \frac{4}{25} \int e^{2x} \cos 5x \, dx$$

$$I = \frac{1}{5} e^{2x} \sin 5x + \frac{2}{25} e^{2x} \cos 5x - \frac{4}{25} I$$

$$I + \frac{4}{25} I = \frac{1}{5} e^{2x} \sin 5x + \frac{2}{25} e^{2x} \cos 5x$$

$$\frac{29}{25} I = \frac{1}{5} e^{2x} \sin 5x + \frac{2}{25} e^{2x} \cos 5x$$

$$\boxed{I = \frac{5}{29} e^{2x} \sin 5x + \frac{2}{29} e^{2x} \cos 5x} + C$$

CHECK: $\frac{d}{dx}\left(\frac{5}{29} e^{2x} \sin 5x + \frac{2}{29} e^{2x} \cos 5x \right)$

$$= \frac{5}{29} \cdot 2e^{2x} \cdot \sin 5x + \frac{5}{29} \cdot e^{2x} \cdot 5\cos 5x + \frac{2}{29} \cdot 2e^{2x} \cdot \cos 5x + \frac{2}{29} e^{2x} \cdot (-5\sin 5x)$$

$$= \frac{10}{29} e^{2x} \sin 5x + \frac{25}{29} e^{2x} \cos 5x + \frac{4}{29} e^{2x} \cos 5x - \frac{10}{29} e^{2x} \sin 5x$$

$$= \frac{29}{29} e^{2x} \cos 5x \quad \checkmark$$