

WRITE YOUR NAME:

MAC 2312 WRITTEN HOMEWORK #6

Due Tuesday February 20th, in Canvas

Question 1. Evaluate the integral.

$$\int_{2/5}^{4/5} \frac{\sqrt{25x^2 - 4}}{x} dx$$

$$\text{Sub } 5x = 2\sec\theta \Rightarrow x = \frac{2}{5}\sec\theta \Rightarrow dx = \frac{2}{5}\sec\theta \tan\theta d\theta$$

$\{25x^2 = 4\sec^2\theta\}$ $\left\{\frac{dx}{d\theta} = \frac{2}{5}\sec\theta \tan\theta\right\}$

$$\text{If } x = \frac{2}{5} \text{ then } \frac{2}{5}\sec\theta = \frac{2}{5} \Rightarrow \sec\theta = 1 \Rightarrow \cos\theta = 1 \Rightarrow \theta = 0$$

$$\text{If } x = \frac{4}{5} \text{ then } \frac{2}{5}\sec\theta = \frac{4}{5} \Rightarrow \sec\theta = 2 \Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$$

$$\int_{x=2/5}^{x=4/5} \frac{\sqrt{25x^2 - 4}}{x} dx = \int_{\theta=0}^{\theta=\pi/3} \frac{\sqrt{4\sec^2\theta - 4}}{\frac{2}{5}\sec\theta} \cdot \frac{2}{5}\sec\theta \tan\theta d\theta$$

$$= \int_{\theta=0}^{\theta=\pi/3} \underbrace{\sqrt{4\tan^2\theta}}_{\text{from } 4(\sec^2\theta - 1)} \cdot \tan\theta d\theta = \int_{\theta=0}^{\theta=\pi/3} 2\tan\theta \tan\theta d\theta$$

$$= 2 \int_0^{\pi/3} \tan^2\theta d\theta = 2 \int_0^{\pi/3} (\sec^2\theta - 1) d\theta = 2 \left[\tan\theta - \theta \right]_0^{\pi/3}$$

$$= 2 \left(\underbrace{\tan\frac{\pi}{3}}_{\sqrt{3}} - \frac{\pi}{3} \right) - 2 \left(\underbrace{\tan 0}_0 - 0 \right) = 2 \left(\sqrt{3} - \frac{\pi}{3} \right)$$

$$\text{OR } \frac{2}{3} (3\sqrt{3} - \pi)$$

Question 2. Evaluate the integral.

$$\int_4^9 \frac{3x+11}{x^2-x-6} dx$$

Denominator x^2-x-6 factors as $(x+2)(x-3)$

$$\frac{3x+11}{(x+2)(x-3)} = \frac{A}{x+2} + \frac{B}{x-3}$$

↓ multiply both sides by $(x+2)(x-3)$

$$3x+11 = A(x-3) + B(x+2) = Ax - 3A + Bx + 2B$$

$$3x+11 = (A+B)x + (-3A+2B) \Rightarrow A+B=3$$

$$-3A+2B=11$$

$$\Rightarrow 3A+3B=9$$

$$-3A+2B=11$$

$$5B=20$$

$$\rightarrow B=4, A=-1$$

$$\int_4^9 \frac{3x+11}{(x+2)(x-3)} dx = \int_4^9 \left(\frac{-1}{x+2} + \frac{4}{x-3} \right) dx$$

$$= \left[-\ln|x+2| + 4\ln|x-3| \right]_4^9$$

$$= \left[\ln|x+2| \right]_4^9 + 4 \left[\ln|x-3| \right]_4^9$$

$$= \ln 6 - \ln 11 + 4(\ln 6 - \underbrace{\ln 1}_0) = 5\ln 6 - \ln 11$$