WRITE YOUR NAME:

MAC 2312 WRITTEN HOMEWORK #7

Due Tuesday March 5th, in Canvas

Question 1. Approximate the integral using Simpson's rule with $n = 4$ and with $n = 6$ subintervals.

$$\int_0^\pi \sin^4 x\,dx \qquad f(x) = (\sin x)^4$$

(i) $n=4 \Rightarrow \Delta x = \frac{\pi - 0}{4} = \frac{\pi}{4} \Rightarrow x_0 = 0, x_1 = \frac{\pi}{4}, x_2 = \frac{\pi}{2}, x_3 = \frac{3\pi}{4}, x_4 = \pi$

$$\frac{\Delta x}{3}\left(f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + f(x_4)\right)$$

$$= \frac{\pi/4}{3}\left(\underbrace{(\sin 0)}_{0}^4 + 4\underbrace{\left(\sin\frac{\pi}{4}\right)}_{1/\sqrt{2}}^4 + 2\underbrace{\left(\sin\frac{\pi}{2}\right)}_{1}^4 + 4\underbrace{\left(\sin\frac{3\pi}{4}\right)}_{1/\sqrt{2}}^4 + \underbrace{(\sin\pi)}_{0}^4\right)$$

$$= \frac{\pi}{12}\left(0 + \underbrace{4\cdot\frac{1}{4}}_{1} + \underbrace{2\cdot 1}_{2} + \underbrace{4\cdot\frac{1}{4}}_{1} + 0\right) = \frac{\pi}{12}\cdot 4 = \boxed{\frac{\pi}{3}}$$

(ii) $n=6 \Rightarrow \Delta x = \frac{\pi}{6} \Rightarrow x_0 = 0, x_1 = \frac{\pi}{6}, x_2 = \frac{\pi}{3}, x_3 = \frac{\pi}{2}, x_4 = \frac{2\pi}{3}, x_5 = \frac{5\pi}{6}, x_6 = \pi$

$$\frac{\Delta x}{3}\left(f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + 2f(x_4) + 4f(x_5) + f(x_6)\right)$$

$$= \frac{\pi/6}{3}\left(\underbrace{(\sin 0)}_{0}^4 + 4\underbrace{\left(\sin\frac{\pi}{6}\right)}_{1/2}^4 + 2\underbrace{\left(\sin\frac{\pi}{3}\right)}_{\sqrt{3}/2}^4 + 4\underbrace{\left(\sin\frac{\pi}{2}\right)}_{1}^4 + 2\underbrace{\left(\sin\frac{2\pi}{3}\right)}_{\sqrt{3}/2}^4 + 4\underbrace{\left(\sin\frac{5\pi}{6}\right)}_{1/2}^4 + \underbrace{(\sin\pi)}_{0}^4\right)$$

$$= \frac{\pi}{18}\left(0 + 4\cdot\frac{1}{16} + 2\cdot\frac{9}{16} + 4\cdot 1 + 2\cdot\frac{9}{16} + 4\cdot\frac{1}{16} + 0\right)$$

$$= \frac{\pi}{18}\left(\frac{2}{8} + \frac{9}{8} + \frac{32}{8} + \frac{9}{8} + \frac{2}{8}\right) = \frac{\pi}{18}\cdot\frac{54}{8} = \boxed{\frac{3\pi}{8}}$$

Question 2. Determine whether the improper integrals converge or diverge, and find their value if they converge.

$$\int_2^\infty \frac{dx}{\sqrt{x}}$$

$$\int_{-\infty}^{-1} \frac{dx}{x^3}$$

(i) Consider $\displaystyle\int_2^M \frac{1}{\sqrt{x}}\,dx = \int_2^M x^{-1/2}\,dx = \left[\frac{x^{1/2}}{1/2}\right]_2^M$

$\displaystyle = 2\left[x^{1/2}\right]_2^M = 2\left(\sqrt{M}-\sqrt{2}\right)$

$\displaystyle\lim_{M\to\infty} 2\left(\sqrt{M}-\sqrt{2}\right) = \infty$, so the integral DIVERGES.

(ii) Consider $\displaystyle\int_M^{-1} \frac{1}{x^3}\,dx = \int_M^{-1} x^{-3}\,dx = \left[\frac{x^{-2}}{-2}\right]_M^{-1}$

$\displaystyle = \left[-\frac{1}{2x^2}\right]_M^{-1} = \left[\frac{1}{2x^2}\right]_{-1}^{M} = \frac{1}{2M^2} - \frac{1}{2}$

$\displaystyle\lim_{M\to\infty}\left(\frac{1}{2M^2} - \frac{1}{2}\right) = 0 - \frac{1}{2} = -\frac{1}{2}$

The integral CONVERGES.