**WRITE YOUR NAME:**

## MAC 2312 WRITTEN HOMEWORK #8

Due Tuesday March 12th, in Canvas

**Question 1.** For each of the following, evaluate the limit or show that it does not exist.

**(a)** $\lim_{n\to\infty} \frac{n^3}{n^4+1}$

**(b)** $\lim_{n\to\infty} \left(\ln(n^3+1) - \ln(3n^3+10n)\right)$ Use $\ln a - \ln b = \ln\left(\frac{a}{b}\right)$

**(c)** $\lim_{n\to\infty} \left(5(-1.01)^n\right)$

**(d)** $\lim_{n\to\infty} \frac{3^n}{3^n+4^n}$

(a) $\lim_{n\to\infty} \frac{n^3}{n^4+1} \cdot \frac{\frac{1}{n^4}}{\frac{1}{n^4}} = \lim_{n\to\infty} \frac{\frac{1}{n}}{1+\frac{1}{n^4}} = \frac{0}{1+0} = 0$

(b) $\lim_{n\to\infty} \ln\left(\frac{n^3+1}{3n^3+10n}\right) = \lim_{n\to\infty} \ln\left(\frac{1+\frac{1}{n^3}}{3+\frac{10}{n^2}}\right) = \ln\left(\frac{1}{3}\right)$ or $-\ln 3$

(c) DOES NOT EXIST because $(-1.01)^n$ gets larger in absolute value (and also oscillates between positive and negative)

(d) $\lim_{n\to\infty} \frac{3^n}{3^n+4^n} \cdot \frac{\frac{1}{4^n}}{\frac{1}{4^n}} = \lim_{n\to\infty} \frac{\left(\frac{3}{4}\right)^n}{\left(\frac{3}{4}\right)^n+1} = \frac{0}{0+1} = 0$

**Question 2.** For each of the following, evaluate the geometric series or show that it diverges.

(a) $1 + \frac{1}{\pi} + \frac{1}{\pi^2} + \frac{1}{\pi^3} + \cdots$

(b) $\sum_{k=3}^{\infty} \frac{3 \cdot 4^k}{7^k}$

(a) Common ratio is $r = \frac{1}{\pi}$, which satisfies $-1 < r < 1$,

so the series CONVERGES.

First term is $a = 1$, so the sum is $\frac{a}{1-r} = \frac{1}{1-\frac{1}{\pi}}$

$= \frac{\pi}{\pi - 1}$.

(b) The first few terms are $\frac{3 \cdot 4^3}{7^3} + \frac{3 \cdot 4^4}{7^4} + \frac{3 \cdot 4^5}{7^5} + \ldots$

The common ratio is $r = \frac{4}{7}$, which satisfies $-1 < r < 1$,

So the series CONVERGES.

The first term is $a = \frac{3 \cdot 4^3}{7^3}$, so the sum is

$$\frac{a}{1-r} = \frac{\frac{3 \cdot 4^3}{7^3}}{1 - \frac{4}{7}} = \frac{3 \cdot 4^3}{7^3} \div \frac{3}{7} = \frac{3 \cdot 4^3}{7^3} \cdot \frac{7}{3} = \frac{4^3}{7^2}$$

or $\frac{64}{49}$