WRITE YOUR NAME:

MAC 2312 WRITTEN HOMEWORK #9

Due Thursday March 21st, in Canvas

Question 1. For each of the following, determine (with reasons) whether the series converges or diverges.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{83}{n}$

(b) $\displaystyle\sum_{n=17}^{\infty} \frac{1}{n}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{83}{n^2}$

(d) $\displaystyle\sum_{n=17}^{\infty} \frac{1}{n^2}$

(a) Constant multiple of the harmonic series $\sum_{n=1}^{\infty} \frac{1}{n}$, which DIVERGES. The given series also DIVERGES.

(b) The harmonic series without its first sixteen terms. DIVERGES.

(c) Constant multiple of $\sum \frac{1}{n^2}$, which is a p-series with $p=2>1$. The series $\sum \frac{1}{n^2}$ converges, so the given series also CONVERGES

(d) This is the series $\sum_{n=1}^{\infty} \frac{1}{n^2}$ without its first sixteen terms.

CONVERGES

Question 2. For each of the following, determine (with reasons) whether the series converges or diverges.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{3n+2}{4n+3}$

(b) $\displaystyle\sum_{n=0}^{\infty} \frac{2^n+n}{3^n}$

(c) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n+\ln n}$

(a) $$\lim_{n\to\infty} \frac{3n+2}{4n+3}\cdot\frac{\frac{1}{n}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{3+\frac{2}{n}}{4+\frac{3}{n}} = \frac{3+0}{4+0} = \frac{3}{4} \neq 0$$

n^{th} term does NOT approach 0, so the series must DIVERGE.

(b) Could use a comparison test. For example, $n < 2^n$.

$$\sum \frac{2^n+n}{3^n} < \sum \frac{2^n+2^n}{3^n} = \sum \frac{2\cdot 2^n}{3^n} \quad \text{geometric with } r = \frac{2}{3}$$

$-1 < r < 1 \Rightarrow$ CONVERGES

Given series is <u>less</u> than a convergent series, so it CONVERGES

(c) Integral test. $\displaystyle\int_2^{\infty} \frac{1}{x(\ln x)^2}\,dx \qquad u = \ln x \Rightarrow du = \frac{1}{x}\,dx$

$$\int_{x=2}^{x=M} \frac{1}{(\ln x)^2}\cdot\frac{1}{x}\,dx = \int_{u=\ln 2}^{u=\ln M} \frac{1}{u^2}\,du = \int_{\ln 2}^{\ln M} u^{-2}\,du = \left[\frac{u^{-1}}{-1}\right]_{\ln 2}^{\ln M}$$

$$= \left[-\frac{1}{u}\right]_{\ln 2}^{\ln M} = \left[\frac{1}{u}\right]_{\ln M}^{\ln 2} = \frac{1}{\ln 2} - \frac{1}{\ln M}$$

$$\int_2^{\infty} \frac{1}{x(\ln x)^2}\,dx = \lim_{M\to\infty}\left(\frac{1}{\ln 2} - \frac{1}{\ln M}\right) = \frac{1}{\ln 2} - 0$$

CONVERGES
So series converges by integral test

(d) Could use a comparison test. For example, $\ln n < n$.

$$\Rightarrow n + \ln n < 2n \Rightarrow \frac{1}{n+\ln n} > \frac{1}{2n}$$

$$\sum \frac{1}{n+\ln n} > \sum \frac{1}{2n} = \sum \frac{1}{2} \cdot \frac{1}{n}$$

constant multiple of harmonic series DIVERGES

Given series is <u>greater</u> than a divergent series, so it DIVERGES.