WRITE YOUR NAME:

MAC 2312 WRITTEN HOMEWORK #10

Due Tuesday March 26th, in Canvas

Question 1. Determine whether the series converges or diverges.

$$\sum_{n=1}^{\infty} \frac{7n^2 - n - 2}{4n^4 - 3n + 1}$$

$a_n = \dfrac{7n^2 - n - 2}{4n^4 - 3n + 1}$ Try limit comparison with $b_n = \dfrac{n^2}{n^4} = \dfrac{1}{n^2}$.

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \left(\frac{7n^2 - n - 2}{4n^4 - 3n + 1} \cdot \frac{n^2}{1} \right) = \lim_{n\to\infty} \frac{7n^4 - n^3 - 2n^2}{4n^4 - 3n + 1}$$

$= \lim_{n\to\infty} \dfrac{7n^4}{4n^4} = \dfrac{7}{4}$ which is not 0 and not ∞.

We conclude that $\sum a_n$ behaves "similarly" to $\sum b_n$.

Since $\sum b_n = \sum \frac{1}{n^2}$ is convergent (p-series with $p = 2 > 1$) then by limit comparison test, we know $\sum a_n$ CONVERGES.

Question 2. Determine whether the series converges or diverges.

$$\sum_{n=0}^{\infty} \frac{(-7)^n}{n!}$$

Try ratio test. $a_n = \dfrac{(-7)^n}{n!}$ $\quad a_{n+1} = \dfrac{(-7)^{n+1}}{(n+1)!}$

$$R = \lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n\to\infty} \left| \frac{(-7)^{n+1}}{(n+1)!} \div \frac{(-7)^n}{n!} \right|$$

$$= \lim_{n\to\infty} \left| \frac{(-7)^{n+1}}{(n+1)!} \cdot \frac{n!}{(-7)^n} \right| = \lim_{n\to\infty} \left| \frac{(-7)^{n+1}}{(-7)^n} \cdot \frac{n!}{(n+1)!} \right|$$

$$= \lim_{n\to\infty} \left| (-7) \cdot \frac{1}{n+1} \right| = \lim_{n\to\infty} \frac{7}{n+1} = 0.$$

Since $R = 0 < 1$, the series CONVERGES (absolutely) by the ratio test.

Question 3. Determine whether the series converges or diverges.

$$\sum_{n=1}^{\infty} \frac{(-n)^3}{3n^3+2}$$

$$a_n = \frac{(-1)^3 n^3}{3n^3+2} = \frac{-n^3}{3n^3+2}$$

Note that $\lim\limits_{n\to\infty} a_n = \lim\limits_{n\to\infty} \frac{-n^3}{3n^3+2} = \lim\limits_{n\to\infty} \frac{-n^3}{3n^3} = \frac{-1}{3} \neq 0$

Since the terms of the series do <u>not</u> approach 0,

the series must DIVERGE.

Question 4. Determine whether the series converges or diverges.

$$\sum_{n=0}^{\infty} \frac{3^{n+4}}{4^{n-2}}$$

$$a_n = \frac{3^n \cdot 3^4}{4^n \cdot 4^{-2}} = \frac{3^4}{4^{-2}} \cdot \left(\frac{3}{4}\right)^n$$

The series is a geometric series with $r = \frac{3}{4}$.

Since $-1 < \frac{3}{4} < 1$, the series CONVERGES