**WRITE YOUR NAME:**

## MAC 2312 WRITTEN HOMEWORK #11

Due Tuesday April 9th, in Canvas

**Question 1.** Find the degree 3 Taylor polynomial centered at 0 for the function $\tan(x)$. Use this to estimate $\tan(0.1)$, and compare this with the answer given by a calculator or computer.

$f(x) = \tan x$

$f'(x) = \sec^2 x$

$f''(x) = 2\sec x \cdot (\sec x)' = 2\sec x \cdot \sec x \tan x = 2\sec^2 x \tan x$

$f'''(x) = 2(\sec^2 x)' \tan x + 2\sec^2 x (\tan x)'$

$= 2 \cdot 2\sec x \cdot \underbrace{(\sec x)'}_{\sec x \tan x} \tan x + 2\sec^2 x \cdot \sec^2 x$

$f(0) = \tan 0 = 0$, $f'(0) = \sec^2 0 = 1$, $f''(0) = 2\sec^2 0 \tan 0 = 0$

$f'''(0) = 4\sec^2 0 \underbrace{\tan^2 0}_{0} + 2\underbrace{\sec^4 0}_{1} = 0 + 2 \cdot 1 = 2.$

Degree 3 Taylor polynomial is $f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3$

$= 0 + \frac{1}{1!}x + \frac{0}{2!}x^2 + \frac{2}{3!}x^3 = x + \frac{x^3}{3}.$

Therefore $\tan(0.1) \approx 0.1 + \frac{(0.1)^3}{3} = 0.1 + \underbrace{\frac{0.001}{3}}_{0.000333\ldots} = 0.100333\ldots$

Calculator says $\tan(0.1) \approx 0.10033467$

**Question 2.** Find the interval of convergence of the power series.

$$\sum_{k=1}^{\infty} \frac{k(x-3)^k}{2^k}$$

$$a_n = \frac{n(x-3)^n}{2^n} \qquad a_{n+1} = \frac{(n+1)(x-3)^{n+1}}{2^{n+1}}$$

Ratio test: $R = \lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n\to\infty} \left| \frac{(n+1)(x-3)^{n+1}}{2^{n+1}} \cdot \frac{2^n}{n(x-3)^n} \right|$

$$= \lim_{n\to\infty} \left| \frac{n+1}{n} \cdot \frac{(x-3)^{n+1}}{(x-3)^n} \cdot \frac{2^n}{2^{n+1}} \right| = \lim_{n\to\infty} \left| \frac{n+1}{n} \cdot (x-3) \cdot \frac{1}{2} \right|$$

$$= \lim_{n\to\infty} \Big( \underbrace{\frac{n+1}{n}}_{\text{This approaches } 1 \text{ as } n\to\infty} \cdot |x-3| \cdot \frac{1}{2} \Big) = 1 \cdot |x-3| \cdot \frac{1}{2} = \underbrace{\frac{|x-3|}{2}}_{\text{This is } R.}$$

We conclude: If $\frac{|x-3|}{2} < 1$ then the series converges.

If $\frac{|x-3|}{2} > 1$ then the series diverges.

The condition $\frac{|x-3|}{2} < 1$ is equivalent to $|x-3| < 2 \Rightarrow -2 < x-3 < 2$
$1 < x < 5$

The series converges if $x$ is in the interval $(1, 5)$.

Note: If $x = 1$ then series $= \sum \frac{k(-2)^k}{2^k} = \sum k(-1)^k$ diverges

If $x = 5$ then series $= \sum \frac{k\,2^k}{2^k} = \sum k$ diverges

**Question 3.** Express the function $f(x) = \ln(1-2x^3)$ as a power series. You may use the power series representations of other known functions.

For example, could start with the series

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}x^n}{n}$$

Replace $x$ with $-2x^3$:

$$\ln(1-2x^3) = -2x^3 - \frac{(-2x^3)^2}{2} + \frac{(-2x^3)^3}{3} - \frac{(-2x^3)^4}{4} + \dots$$

$$= -2x^3 - \frac{4x^6}{2} - \frac{8x^9}{3} - \frac{16x^{12}}{4} - \dots$$

$$= \sum_{n=1}^{\infty} \frac{-2^n x^{3n}}{n}$$