

WRITE YOUR NAME:

MAC 2312 WRITTEN HOMEWORK #12

Due Tuesday April 16th, in Canvas

Question 1. Find the degree 3 Taylor polynomial centered at 0 for the function $\arcsin(x)$. Use this to estimate $\arcsin(0.1)$, and compare this with the answer given by a calculator or computer.

$$f(x) = \arcsin x$$

$$f'(x) = \frac{1}{\sqrt{1-x^2}} = (1-x^2)^{-1/2}$$

$$f''(x) = -\frac{1}{2} (1-x^2)^{-3/2} \cdot (-2x) = x (1-x^2)^{-3/2}$$

$$f'''(x) = 1 \cdot (1-x^2)^{-3/2} + x \cdot \frac{-3}{2} (1-x^2)^{-5/2} \cdot (-2x)$$

$$f(0) = 0, f'(0) = 1, f''(0) = 0, f'''(0) = 1 \cdot 1 + 0 = 1$$

$$\begin{aligned} \text{Degree 3 Taylor polynomial is } & 0 + \frac{1}{1!}x + \frac{0}{2!}x^2 + \frac{1}{3!}x^3 \\ & = x + \frac{x^3}{6} \end{aligned}$$

$$\begin{aligned} \arcsin(0.1) &\approx 0.1 + \frac{(0.1)^3}{6} = 0.1 + \frac{0.001}{6} \\ &= 0.1 + 0.00016666\dots \\ &= 0.10016666\dots \end{aligned}$$

Calculator says $\arcsin(0.1) \approx 0.1001674\dots$

Question 2. Evaluate the limit using Taylor series.

$$\lim_{x \rightarrow 0} \frac{3 \arcsin x - 3x}{x^3}$$

$$\lim_{x \rightarrow 0} \frac{3 \left(x + \frac{x^3}{6} + (\text{terms containing } x^4 \text{ or higher}) \right) - 3x}{x^3}$$

$$= \lim_{x \rightarrow 0} \frac{3x + \frac{x^3}{2} + (\text{terms containing } x^4 \text{ or higher}) - 3x}{x^3}$$

$$= \lim_{x \rightarrow 0} \frac{\frac{x^3}{2} + (\text{terms containing } x^4 \text{ or higher})}{x^3}$$

$$= \lim_{x \rightarrow 0} \left(\frac{1}{2} + (\text{terms containing } x, x^2, \text{etc.}) \right)$$

$$= \frac{1}{2} + 0 = \frac{1}{2}$$

Note: If you're curious how to write the above with fewer words, you can look up "big O" notation